A

MANUAL

OF

ARITHMETIC,

ADAPTED FOR THE USE OF SCHOOLS, PRIVATE TUTORS, AND FAMILIES.

BY GEORGE HUTTON,

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IN KING'S COLLEGE SCHOOL.

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TO

THE REV. JOHN RICHARDSON MAJOR, D.D.,

HEAD MASTER

OF

KING'S COLLEGE SCHOOL,

THIS

MANUAL OF ARITHMETIC

IS

MOST RESPECTFULLY DEDICATED BY HIS GRATEFUL AND OBEDIENT SERVANT,

THE AUTHOR.

Jan. 1, 1844.

PREFACE.

The study of Arithmetic as a science is so obviously calculated to develope the reasoning faculties, to call forth the energies of the mind, and impart a quickness of perception and an accuracy of judgment, that it cannot be too strongly recommended to young persons, as a successful introduction to the pursuit, and a powerful auxiliary in the attainment, of all valuable knowledge.

But obvious as is the tendency of a right study of this initiatory science, and salutary as are its influences upon the understanding, it has been too much the practice to teach it to young persons merely as a series of dry formulæ for obtaining mechanically the practical result of principles that have not been previously demonstrated, and thus a most important opportunity of cultivating the mind is irretrievably lost.

In the hope of supplying in some degree what the Author cannot but regard as a deficiency in the system of teaching Arithmetic, the present work is designed, by an ample development and a familiar illustration of elementary principles, to lead the pupil progressively into the exercise of his reasoning powers, and enable him to derive, through his own judgment, simple and efficient rules for the solution of its most intricate problems.

For this purpose, it regards every possible question in Arithmetic, as proposing nothing more, than either the increase or the decrease of a given quantity whose

required magnitude is dependent on the increase or decrease of other quantities with which it may be connected by the several conditions in the question proposed.

With this view, it fully explains to the pupil, first, the nature of dependent quantities and the law of their variation, from his knowledge of which he will instantly perceive whether the magnitude of the given quantity is to be increased or decreased; and secondly, the doctrine of Ratio, from which he will, with equal facility, ascertain the precise extent to which the required increase or decrease is to be made.

From these two simple and obvious principles of Variation and Ratio, which are the elements of Proportion, and perfectly within his apprehension, the pupil will easily derive a more direct, simple, and efficient rule for the solution even of the most difficult questions, than he can possibly acquire through his memory alone.

He will perceive that whatever may be the number of conditions contained in the question, each of them is expressed by a ratio which distinctly shows both the cause and the extent of the increase or decrease required by that particular condition, in the magnitude of the given quantity, which must always vary either directly or inversely, as the quantities of each ratio.

And, lastly, reducing to their lowest terms, and compounding these several ratios according to the relation which the given quantity may have to each of them separately considered, he will obtain one single ratio combining all their influences, and which will precisely determine the required magnitude of the given quantity, and afford a clear and satisfactory solution of the question.

CONTENTS.

EXPLANATION

OF

SIGNS USED IN THIS WORK.

Char.

$=$ The sign of Equality, denoting that the quantities between which it is placed are equal to each other.

$+$ Plus or more, the sign of Addition, denoting that the quantities between which it is placed are to be added together; as $9+6=15$.

$-$ Minus or less, the sign of Subtraction, denoting that the latter of the quantities between which it is placed is to be subtracted from the former; as $9-6=3$.

$\times$ The sign of Multiplication, denoting that the quantities between which it is placed are to be multiplied together; as $9\times 6=54$.

$\div$ The sign of Division, denoting that the former of the two quantities between which it is placed is to be divided by the latter; as $12\div 4=3$.

$:$ The sign of Ratio, denoting the relative magnitude which the former of the quantities between which it is placed has in comparison with the latter.

$::$ The sign of Equality of Ratios, denoting that the ratio of two quantities which precede is equal to the ratio of the two which follow.

Thus, $9 : 3 :: 18 : 6$, denotes that the ratio $9 : 3$ is equal to the ratio $18 : 6$, and consequently that the four quantities taken in that order are proportionals.

TABLES OF WEIGHTS AND MEASURES.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

TROY WEIGHT.

4 grains	make	1 carat
24 grains	—	1 pennyweight, dwt.
20 pennyweights	—	1 ounce, oz.
12 ounces	—	1 pound, lb.

AVOIRDUPOIS WEIGHT.

16 drams, drs.	make	1 ounce, oz.
16 ounces	—	1 pound, lb.
14 pounds	—	1 stone
28 pounds	—	1 quarter, qr.
4 quarters	—	1 hundred weight, cwt.
20 hundred weight	—	1 ton

APOTHECARIES' WEIGHT.

20	grains	make	1 scruple, sc. or ℈
3	scruples	—	1 dram, dr.
8	drams	—	1 ounce, ℥
12	ounces	—	1 pound

WOOL WEIGHT.

7	pounds	make	1 clove
2	cloves	—	1 stone
2	stones	—	1 tod
6½	tods	—	1 wey
2	weys	—	1 sack
12	sacks	—	1 last
240	pounds	—	1 pack

HAY AND STRAW WEIGHT.

36	pounds	make	1 truss straw
56	pounds	—	1 truss old hay
60	pounds	—	1 truss new hay
36	trusses	—	1 load

DRY MEASURE.

4	gills	make	1 pint
2	pints	—	1 quart
2	quarts	—	1 pottle
4	quarts	—	1 gallon
2	gallons	—	1 peck
4	pecks	—	1 bushel
4	bushels	—	1 coomb
8	bushels	—	1 quarter
5	quarters	—	1 wey
2	weys	—	1 last

COAL MEASURE.

4	pecks	make	1 bushel
3	bushels	—	1 sack
3	sacks	—	1 vat or strike
12	sacks	—	1 chaldron
21	chaldrons	—	1 score

LIQUID MEASURE.

4	gills	make	1 pint
2	pints	—	1 quart
4	quarts	—	1 gallon
9	gallons	—	1 firkin beer
10	gallons	—	1 anker spirits
18	gallons	—	1 kilderkin
36	gallons	—	1 barrel beer
42	gallons	—	1 tierce wine
54	gallons	—	1 hogshead beer, hhd.
63	gallons	—	1 hogshead wine
2	hogsheads	—	1 pipe wine
2	pipes	—	1 tun

CLOTH MEASURE.

2¼	inches	make	1 nail
4	nails	—	1 quarter of a yard
3	quarters	—	1 Flem. ell
4	quarters	—	1 yard
5	quarters	—	1 English ell
6	quarters	—	1 French ell

LONG MEASURE.

3	barleycorns	make	1 inch
12	inches	—	1 foot
3	feet	—	1 yard
6	feet	—	1 fathom
5½	yards	—	1 pole or perch
40	poles	—	1 furlong
8	furlongs	—	1 mile
3	miles	—	1 league
60 69½	geographical or statute miles	—	1 degree

SQUARE OR SUPERFICIAL MEASURE.

144	inches	make	1 square foot
9	square feet	—	1 square yard
30¼	square yards	—	1 square pole
40	square poles	—	1 rood
4	roods	—	1 acre
640	acres	—	1 square mile

CUBIC OR SOLID MEASURE.

1728 cubic inches	make	1 cubic foot
27 cubic feet	—	1 cubic yard
40 cubic feet of rough or 50 cubic feet of hewn timber	—	1 load
277·274 cubic inches	—	1 imperial gallon

TIME.

60 seconds	make	1 minute
60 minutes	—	1 hour
24 hours	—	1 day
7 days	—	1 week
4 weeks	—	1 month
12 calendar months or 13 common months 1 day and 6 hours, or 365 days 6 hours	—	1 year

PRACTICE TABLE.

Parts of a Shilling.

d.		
$\frac{3}{4}$	equal to	$\frac{1}{16}$
$1\frac{1}{2}$	—	$\frac{1}{8}$
3	—	$\frac{1}{4}$
4	—	$\frac{1}{3}$
6	—	$\frac{1}{2}$

Parts of a Pound.

s.	*d.*		
1	3	equal to	$\frac{1}{16}$
1	8	—	$\frac{1}{12}$
2	6	—	$\frac{1}{8}$
3	4	—	$\frac{1}{6}$
6	8	—	$\frac{1}{3}$

Parts of a Quarter.

lb.		
$3\frac{1}{2}$	equal to	$\frac{1}{8}$
4	—	$\frac{1}{7}$
7	—	$\frac{1}{4}$
14	—	$\frac{1}{2}$

Parts of a Cwt.

lb.		
7	equal to	$\frac{1}{16}$
8	—	$\frac{1}{14}$
14	—	$\frac{1}{8}$
16	—	$\frac{1}{7}$

ARITHMETIC.

1. **Arithmetic** is both an art and a science.

2. As an art, Arithmetic teaches the proper expression and arrangement of numbers with reference to their magnitude, increase, or decrease; and comprehends the system of Numeration, and the rules of Addition and Subtraction, with their modifications, Multiplication and Division.

3. As a science, Arithmetic is that branch of the Mathematics which treats of the several relations, in respect of magnitude and variation, that subsist between all such quantities as can be represented by numbers, and is founded entirely upon the doctrine of ratio and proportion.

NUMERATION.

1. **Numeration** is the art of expressing numbers either in figures or by words.

2. Numbers are expressed in writing by the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, and the cipher 0.

3. The cipher has no value in itself, neither can it by itself express any number whatever; it is used only in adjusting the local values of the other figures.

4. Each of the other figures, which are called digits, points out by itself a definite number of units, or ones, which is denoted by its form, and is called its simple value.

Thus, the figure 1 points out a single unit, or one entire thing of any kind; as one bushel, one mile, one hour; the figure 2 points out two entire things of any kind; as two bushels, two miles, two hours, &c.

5. The greatest of these figures, 9, cannot by itself express more than nine units; therefore, to express any number greater than 9, we give to each of the nine digits an additional value, arising from the place in which it is written, and which is therefore called its local value.

6. This local value increases the simple value of the

digit ten times for every removal towards the left hand from the first place, or place of units, in which it would appear if written by itself; and these two values combined express the entire magnitude of the number it is intended to represent.

Thus, by placing a cipher 0 on the right hand of the digit 1, it is removed from the first into the second place, 10, where its value is ten times as great as in the first place, and it expresses ten ones, or ten.

By placing two ciphers on the right hand of the digit 1, it is removed into the third place, 100, in which its value is ten times as great as in the second place, or a hundred times as great as in the first place, and it expresses ten tens, or one hundred.

By placing three ciphers on the right hand of the digit 1, it is removed into the fourth place, 1000, in which its value is ten times as great as in the third place, or a thousand times as great as in the first place, and it expresses ten hundreds, or one thousand.

By placing four ciphers on the right hand of the digit 1, it is removed into the fifth place, 10000, in which its value is ten times as great as in the fourth place, or ten thousand times as great as in the first place, and it expresses ten thousands.

And by placing five ciphers on the right hand of the digit 1, it is removed into the sixth place, 100000, in which its value is ten times as great as in the fifth place, or a hundred thousand times as great as in the first place, and it expresses ten tens of thousands, or one hundred thousand.

7. Hence the use of the cipher in adjusting the local values of the other figures; for the ciphers, having no values in themselves, serve only as indices to the place in which the digit is written, and from which it derives its local value.

Thus, the local value of a figure, when written in the

1st place, as			 1,	is	units.
2nd	,,	1×10	. . 10,		tens.
3rd	,,	10×10	. . 100,		hundreds.
4th	,,	100×10	. 1000,		thousands.
5th	,,	1000×10	. 10000,		tens of thousands.
6th	,,	10000×10	. 100000,		hundreds of thousands.

These six places, with their several local values, form what is called a period.

8. The greatest number that can be written in one period is a series of nines, 999999; which, beginning at the left hand or highest place of the period, severally express 9 hundred thousands, 9 tens of thousands, 9 thousands, 9 hundreds, 9 tens, and 9 units, or collectively nine hundred and ninety-nine thousand nine hundred and ninety-nine—the word units being always implied, unless some other name is expressed.

9. If one unit more be added to this number, it will amount to ten hundred thousands, which number is called a million, and is expressed by the digit 1 with six ciphers on the right hand; but these six ciphers, which in themselves express no value, by occupying the six places of the first period, serve only as indices to show that the digit 1 is in the first place of the second period, 1,000000, where it expresses one million.

10. The number of periods is unlimited, for as often as the simple and local values in any period amount to ten hundred thousands, that number will be expressed by the digit 1, written in the first place of the next higher period, ten hundred thousand or a million of units in any one period always making one unit in the next higher period, as in the following scheme:—

Units,	999999
	1
Millions,	1,000000
	999999
Billions,	1,000000,000000
	999999
Trillions,	1,000000,000000,000000
	999999
Quadrillions,	1,000000,000000,000000,000000
	999999
Quintillions,	1,000000,000000,000000,000000,000000
	999999
Sextillions,	1,000000,000000,000000,000000,000000,000000

11. Thus any number, however great, may be properly expressed in writing by selecting such of the digits as by their form will point out the simple values, and writing them in such places of the period as will denote the local values contained in that number.

Ex.—To express in figures the number three hundred thousand four hundred and five millions.

This number contains three hundred thousand, which has the simple value three, and the local value hundreds of thousands; we therefore select the digit 3 to point out the simple value, and write it in the sixth place of the period to denote the local value of this part of the number 300000.

Secondly, it contains four hundreds, which has the simple value four, and the local value hundreds; we therefore select the digit 4 to point out the simple value, and write it in the third place of the period to denote the local value of this part of the number 400.

Thirdly, this number contains five, which has the simple value five, and the local value units; we therefore select the digit 5 to point out the simple value, and write it in the first place of the period to denote the local value of this part of the number 5.

Thus, all the simple and local values contained in the number three hundred thousand four hundred and five are properly expressed by the digits 3, 4, and 5, written respectively in the sixth, third, and first places of the period 300405; and the proper relation of these several places is preserved by writing ciphers in the fifth, fourth, and second places, in which no values are expressed.

Lastly, as all these simple and local values are to express so many millions, we place six ciphers on the right hand of the period in which they are written, to remove them into the second period, 300405,000000, where they justly express the given number, three hundred thousand four hundred and five millions.

TO EXPRESS WRITTEN NUMBERS IN WORDS.

12. The true value of every figure in a number is ascertained by considering in what period and in what place of the period it is written, and is properly expressed

in words by adding the simple value of the figure to its local value, and to both these the name of the period.

Thus, to express in words the number 300405,000000.

Here, beginning at the highest place, we find the figure 3 in the sixth place of the second period; it has therefore the simple value three, and the local value hundreds of thousands; and to these adding the name of the second period, millions, we read for this part of the number three hundred thousand millions.

Secondly, we find the next lower figure 4 in the third place of the second period; it has therefore the simple value four, and the local value hundreds; and to these adding the name of the period, we read for this part of the number four hundred millions.

Lastly, we find the next lower figure 5 in the first place of the second period; it has therefore the simple value five and the local value units; and to these adding the name of the period, we read for this part of the number five millions.

Hence the whole of the number 300405,000000, properly expressed in words, is three hundred thousand millions, four hundred millions, and five millions; or more simply, three hundred thousand four hundred and five millions.

13. The most accurate and expeditious method of reading a large number is to separate it into distinct periods, by marking off with a comma every six figures from the right hand, and beginning at the highest period to read the simple and local values in each period separately, pronouncing after each series the name of the period in which it is written.

Ex.—To read the number 123,406005,008934.

Having, as above, separated the number into distinct periods, we read, in the highest period, one hundred and twenty-three billions; in the next lower, four hundred and six thousand and five millions; and in the lowest, eight thousand nine hundred and thirty-four.

Thus the whole number, properly expressed in words, is one hundred and twenty-three billions, four hundred and six thousand and five millions, eight thousand nine hundred and thirty-four.

USE OF THE CIPHER.

15. The placing of ciphers on the right hand of any number increases its value ten times for every cipher so placed.

For if one cipher be placed on the right hand of any number, it removes the figure which was previously in the first place into the second, that which was in the second place into the third, and so on throughout the whole series. Hence every figure in the number being removed one place higher is increased ten times, and consequently the whole number is increased ten times.

If two ciphers be placed on the right hand of any number, every figure will be removed two places higher and increased a hundred times, and consequently the whole number will be increased a hundred times, &c., &c.

16. The cutting off or taking away of ciphers from the right hand of any number decreases its value ten times for every cipher so taken away.

For if one cipher be taken away, the figure which was previously in the second place is removed into the first, that which was in the third place into the second, and so on throughout the whole series: hence every figure in the number being removed one place lower is ten times decreased, and consequently the whole number is ten times decreased.

If two ciphers be taken away, every figure in the number will be removed two places lower, and the whole number will be decreased a hundred times.

From these considerations is derived the use of the cipher in abridging the operations of Multiplication and Division.

EXERCISES IN NUMERATION.

1. In what period, and in what place of it, must the figure 9 be written to express nine hundred millions?

2. In what period, and in what places, must the figures 3 and 6 be written to express three thousand and sixty billions?

3. Where must the figures 5, 6, and 7 stand to express the values fifty thousand and sixty-seven trillions?

4. In what periods, and in what places, must the figures 6, 7, 8, 9, and 3, stand to express the values six hundred

quadrillions, seventy-eight thousand billions, ninety thousand and thirty?

5. In what periods, and in what places of them, must the figures 7, 6, 3, 8, and 9 stand to express the values seven hundred thousand and sixty-three quintillions, eight hundred and ninety millions?

6. In what periods and places must the nine figures stand to express one hundred thousand and two quintillions, three thousand and four trillions, five hundred thousand and six billions, seven hundred and eighty thousand and nine?

7. How many ciphers must be placed on the right hand of the figure 7 to make it ten million times greater?

8. How many ciphers must be added to the figure 3 to make it express three hundred thousand trillions?

9. How many ciphers must be placed on the right hand of the figure 5 to make it a hundred thousand times greater?

10. How many ciphers must be taken away from the number three hundred thousand quadrillions to make it express thirty thousand billions?

11. How many ciphers must be cut off from the right hand of a number to decrease its value a hundred thousand times?

12. How many ciphers must be written between the values seven hundred trillions and four thousand millions to preserve the proper relations of the periods?

13. How many ciphers must be written, for the same purpose, between the values nine hundred thousand quadrillions and seventy thousand billions?

14. Express in figures the number nine hundred thousand millions, seven hundred and eleven.

15. Write in figures the number three hundred and eleven thousand quintillions, nine thousand trillions, thirty thousand and seven.

16. Express in figures the number eleven thousand quadrillions, three hundred and nineteen trillions, fourteen

thousand five hundred billions, six hundred thousand and seventy millions, three thousand and four?

Express in words the following numbers:—

17. 160746238.

18. 27630004756.

19. 300706767016074.

20. 2160760043670600076.

21. 3160476091119220076307l.

22. 700760470760476076276981l.

23. 76033577420041067200756132.

24. 12345678909876543210123456789.

ADDITION.

1. Addition is the collecting of several numbers, representing quantities of the same kind, into one number, which shall be equal to the amount of their united values, and consequently express their sum.

Thus, if the numbers 3 and 5 both represent units, or both represent pence, they may be collected into one number, $3+5=8$; for 8 units are evidently the sum of 3 units and 5 units, and 8 pence the sum of 3 pence and 5 pence.

2. Numbers which do not represent quantities of the same kind cannot be collected into one number, for the sum of the numbers cannot be equal to the amount of their united values.

Thus 3 tens and 5 units, or 3 shillings and 5 pence, cannot be collected into the one number, $3+5=8$; for 8, whether considered as 8 tens or as 8 units, cannot be equal to the sum of 3 tens and 5 units; neither can 8, whether considered as 8 shillings or as 8 pence, be equal to the sum of 3 shillings and 5 pence.

3. The only expression to be found for the united values of these quantities is 3 tens and 5 units, or 3 shillings and 5 pence, in which the addition consists solely in the word *and*, which joins them together as unlike quantities, but cannot incorporate them into one number, or add them into one sum.

4. Hence, when we have to add together several numbers, consisting either of different local values, as hundreds, tens, and units in simple addition, or of pounds, shillings, and pence in compound addition, it is evident that those parts only in each which are of the same kind can be collected into one number or added into one sum.

5. It is equally evident, in either case, that the amount will consist of as many different sums as there are either

different local values or different denominations in the numbers added; but as these several sums are taken collectively, as the total amount of the several quantities, this amount may be called their sum.

The rule, therefore, for addition, whether of simple numbers or of compound quantities, is general, and is obviously deduced from the principle laid down.

RULE.

1. Place the numbers to be added in such order that the several local values or denominations in each, which are of the same kind, may stand directly under each other.

2. Add together all the numbers in the lowest place or denomination, and find how many of the next higher rank are contained in their sum, and also how many remain.

3. Write down this remainder, or if there be no remainder write a cipher under the numbers already added, and add the number of the next higher rank to the numbers in the next higher place or denomination.

4. Proceed in this manner to add the numbers in every succeeding place or denomination, and the several amounts taken together will be the whole amount or sum of the given numbers.

Ex.—To add $4567+678+53+9$.

$$\begin{array}{r} 4567 \\ 678 \\ 53 \\ 9 \\ \hline 5307 \\ \hline \end{array}$$

1. Here, placing the given numbers as directed, we find the sum of the units to be 27, which contains 2 of the next higher rank, tens, with 7 units remaining. We therefore write 7 under the line of units, and add the 2 tens to the numbers in the next higher place. We next find the sum of the tens, including the 2 carried from the place of units, to be 20, which contains 2 of the next higher rank or hundreds, without any remainder. We therefore write a cipher under the line of tens, and add the 2 hundreds to the numbers in the next higher place. We next find the sum of the hundreds, including the 2 carried from the place of tens, to be 13, which contains 1 of the next higher rank or thousands, with 3 hundreds remaining. We therefore write 3 under the line of hundreds, and add the thousand to the numbers in the next higher place, and find the sum of the thousands, including the 1

carried, to be 5, which, as it contains none of a higher rank, we write down under the line of thousands; and these several amounts taken together are the whole amount or sum of the given numbers.

Ex.—2. To add 15*l.* 16*s.* 8*d.*+3*l.* 17*s.* 2*d.*+18*s.* $11\frac{1}{2}$*d.* + $9\frac{3}{4}$*d.*

Here, placing the given quantities as directed, we find the sum of the farthings to be 5, which contains 1 penny, and 1 farthing remaining. We therefore write 1 under the line of farthings, and add the 1 penny to the numbers in the next higher denomination. We next find the sum of the pence, including the 1 carried from the farthings, to be 31, which contains 2 shillings, with 7 pence remaining. We therefore write 7 under the line of pence, and add the 2 shillings to the numbers in the next higher denomination. We next find the sum of the shillings, including the 2 carried from the pence, to be 53, which contains 2*l.*, with 13 shillings remaining. We therefore write 13 under the line of shillings, and add the 2*l.* to the numbers in the next higher denomination, of which we lastly find the sum, including the 2 carried from the shillings, to be 20, which we write under the line of pounds; and these several amounts taken together are the whole amount, 20*l.* 13*s.* $7\frac{1}{4}$*d.*, the sum of the given quantities.

£.	*s.*	*d.*
15	16	8
3	17	2
0	18	$11\frac{1}{2}$
0	0	$9\frac{3}{4}$
20	13	$7\frac{1}{4}$

EXAMPLES FOR PRACTICE.

1. Add the quantities 39+9+25+317+30+1+25+3064.

2. Add the numbers 30765+216+3674+20+119+9+215+67.

3. 123456+2007+3164+10076+379+2679+369++16.

4. Find the amount of the quantities, three hundred thousand and seven+fifteen millions and seventy-nine+ninety-nine thousand and nineteen+seventy-eight+thirteen thousand eight hundred and thirteen+seventy thousand and seventeen+sixty-five millions+six hundred and eleven.

5. Add 300042 + 56000342 + 700 + 1732 + 71876 + 98768532 + 30176 + 26.

6. Add 786532 + 2169 + 4 + 716265 + 8752 + 17 + 6875 + 3689.

7. Add 760432 + 16076 + 26358 + 239 + 6754 + 987 + 89 + 6 + 19.

8. Add thirty millions + fifty-five millions + seventy-nine thousands + five hundred and eighteen thousands + six hundred and ninety-seven millions + eighteen thousand millions + seventy-seven thousands + six hundred and fifty-four thousand, seven hundred and eighty-one.

9. Add 17 + 367 + 26875 + 6935 + 716 + 38976478 + 678635875 + 39769.

10. Add 310000 + 700 + 715000 + 6970 + 35760 + 2060 + 78976532 + 98706.

11. Add 937626576 + 268706576 + 30706708 + 2067 + 367426 + 37 + 206748 + 31647658347.

12. Add seven thousand billions and nine hundred thousand + eighty-five thousand millions and seventeen + six hundred thousand millions and seventy-nine + fifteen millions + fifteen thousand seven hundred and eighteen + ninety-nine billions, nine hundred millions, nine hundred and nine.

N.B. Other examples may be easily supplied.

MULTIPLICATION.

1. Multiplication is the repeating of a given number or quantity any proposed number of times, and finding its amount when so many times repeated; and is, consequently, only a more compendious method of Addition, when the numbers to be added are all equal.

In Multiplication, three things are to be considered:—

1. The multiplicand, which is the number to be repeated.
2. The multiplier, which shows how many times the multiplicand is to be repeated.
3. The product, which shows the amount of the multiplicand when so many times repeated.

2. In Multiplication every single unit in the multiplicand must be made equal to the whole number of units in the multiplier.

Hence, however inappropriate the term, the product may be either equal to, or greater, or less than the multiplicand.

3. If the multiplier be a single unit, as 1, the product will be equal to the multiplicand; for every unit in the multiplicand being equal to 1, the multiplicand will remain unaltered: thus, $24 \times 1 = 24$

4. If the multiplier be greater than a single unit, as 2, the product will be twice as great as the multiplicand; for every unit in the multiplicand will be made equal to two units: thus, $24 \times 2 = 48$.

5. If the multiplier be less than a single unit, as $\frac{1}{2}$, the product will be twice as small as the multiplicand; for every unit in the multiplicand will be made equal to half a unit: thus, $24 \times \frac{1}{2} = 12$.

6. But while the multiplier is a whole number greater than 1, we may say that to multiply a given number is to make it a certain number of times greater.

7. A number may be made greater, either by increasing its simple values, or by increasing its local values.

8. The simple value of a number is increased by Addition. Thus, to make the number 2 three times as great, that number may be added to itself twice, or three of that number may be added together; thus, $2+2+2=6$, which is evidently equal to three times 2, and consequently three times as great.

9. The local value of a number is increased merely by position; as, to increase the local value of the number 2 ten times, we place a cipher on its right hand, which removes it from the first place into the second, 20, in which position it is equal to 2 tens, and consequently its local value is ten times increased.

10. When the multiplier has only a simple value, as is always the case when it is less than 10, it affects only the simple values of the multiplicand; thus, 2 units $\times 3=6$ units, 2 tens $\times 3=6$ tens, in which only the simple value of the multiplicand is increased, and this increase arises solely from Addition.

But instead of actually performing these additions, we remember, from the Multiplication Table, the several amounts of the nine digits when repeated so many times, and instantly find the amounts of the several figures in the multiplicand.

11. When the multiplier has only a local value, as is always the case when it consists of the digit 1 with ciphers on the right hand, as 10, 100, 1000, &c., it affects only the local values of the multiplicand; as 2 units $\times 10=2$ tens $=20$; 2 tens $\times 10=2$ hundreds $=200$. In this case the local value of the multiplicand only is increased, and this increase arises solely from position.

12. When the multiplier has both a simple and a local value, as 30, 300, 3000, &c., it affects both the simple and the local values of the multiplicand, as 2 units $\times$ 3 tens $=$ 6 tens or 60; in which instance the simple value 2 is increased to 6 by addition, and the local value, 6 units, is increased to 6 tens $=60$ by position.

13. Hence, to multiply a given number any proposed number of times, we have only to increase its simple

values by Addition as many times as is denoted by the simple values of the multiplier, and also its local values by position as many times as is denoted by the local values of the multiplier; and beginning at the lowest place of the multiplicand, and with the lowest figure in the multiplier, to continue this process till every figure in the multiplier has been used.

Thus, to multiply any number by 134, we first make each of the figures in the multiplicand 4 times as great by Addition, writing down under each the units in its amount, or if no units a cipher, and adding the tens as so many units to the product of the figure in the next higher place; and thus make the whole multiplicand 4 times as great, which is the whole increase required by this figure of the multiplier, which has only the simple value 4.

2ndly. To multiply by the figure 3, which has the local value tens, we first make the multiplicand 3 times as great by addition for the simple value of this figure, and then ten times greater by position for its local value, by writing the first figure of this product in the second place, or place of tens; thus making the multiplicand 30 times as great, which is the whole increase required both by the simple and local values of this figure of the multiplier.

3rdly. To multiply by the figure 1, which has only the local value hundreds: as this multiplier does not affect the simple values, we have only to make the local values of the multiplicand 100 times as great by position for the local value of the figure 1, by writing the first figure of the product in the third place, or place of hundreds; thus making the multiplicand 100 times as great, which is the whole increase required by this figure of the multiplier.

Here it is obvious that the first line of products, of which the first figure is in the place of units, is the whole product of the multiplicand multiplied by the figure 4, and consequently 4 times as great; that the second line of products, of which the first figure is in the place of tens, is the whole product of the multiplicand multiplied by the figure 3=30, and consequently 30 times as great; and that the third line of products, of which the first figure is in the place of hundreds, is the whole product of the multiplicand multiplied by the figure 1=100, and consequently 100 times as great.

Lastly, it is evident that if these three lines of products be added together, their sum will be the whole product of the multiplicand multiplied by the whole multiplier, 134, and consequently 134 times as great; which is the whole increase required by the simple and local values of all the several figures of the multiplier.

Ex.—To multiply 567 by 134.

$$\begin{array}{lrr}
 & 567 & \\
 & \underline{134} & \\
1.\ 567\times4=2268 & 2268= & 4 \text{ times.} \\
2.\ 567\times3=1701;\ 1701\times10=17010= & 1701\ = & 30 \text{ times.} \\
3.\ 567\times1=567;\ 567\times100=56700= & 567\ \ = & \underline{100} \text{ times.} \\
2268+17010+56700\ \ = & \overline{75978}= & 134 \text{ times.}
\end{array}$$

RULE.

1st. Write the multiplier under the multiplicand in such order that units may stand under units, tens under tens, &c., as in Addition.

2nd. Begin at the place of units, and multiply every figure in the multiplicand by each of the figures in the multiplier successively, writing down under each the units in its product, and adding the tens as so many units to the product of the figure in the next higher place.

3rd. Continue this process till you have multiplied by all the figures of the multiplier, taking care always to write the first figure of every line of products where it may have the same local value as the figure by which you are multiplying.

4th. And lastly, add together the several lines of products in the order in which they are written, and their sum will be the whole product of the multiplicand multiplied by the whole multiplier.

It may be observed that the writing of the first figure of the second line of products in the second place is equivalent to the placing of a cipher on its right hand; but as the first figure of the first line of products is always in the place of units, it is a sufficient guide to the local values of all the other products, and therefore the ciphers are omitted.

14. The multiplier may sometimes be considered as the product of two or more single figures; as, for instance, 24 may be considered as the product of $4\times6=24$; and 192 as the product of the several single figures, $4\times6\times8=192$.

In this case we separate the multiplier into its several component parts, and multiply the multiplicand by any one of these parts, and every resulting product by another of them, till all have been used; and the last product will be the whole product of the original multiplicand, multiplied by the whole of the multiplier.

Ex.—To multiply 365 by 24.

```
 365
   4
----
1460=4 times.
   6
----
8760=4×6=24 times.
----
```

Here separating the multiplier 24 into its component parts, $4\times6=24$, we first multiply the multiplicand 365 by 4, and obtain the product 1460, which is 4 times as great as the multiplicand; and now multiplying this product 1460 by 6, the other component part of the multiplier, we obtain the product 8760, which is 6 times as great as the product 1460, or $4\times6=24$ times as great as the original multiplicand 365, and consequently the whole product of 365 multiplied by the whole of the multiplier 24.

Ex. 2.—To multiply 365 by 192.

```
  365
    4
 ----
 1460=4 times.
    6
 ----
 8760=4×6=24 times.
    8
-----
70080=4×6×8=192 times.
-----
```

Here separating the multiplier 192 into its component parts, $4\times6\times8=192$, we multiply the multiplicand 365 by 4, and the resulting product 1460 by 6, as in the former example, and obtain the product 8760, which is 24 times as great as the multiplicand 365; and now multiplying this last product by the last of the component parts 8, we obtain the product 70080, which is 8 times as great as the product 8760, $6\times8=48$ times as great as the preceding product 1460, and $4\times6\times8=192$ times as great as the

original multiplicand 365, and consequently the whole product of the multiplicand 365 multiplied by the whole of the multiplier 192.

This method of resolving the multiplier into its several component parts is chiefly of use in the multiplication of compound quantities, consisting of different denominations not bearing to each other a constant tenfold relation as in simple numbers, and of which the several products cannot have their local values expressed by position, as in Simple Multiplication.

But it is not always practicable to separate a large multiplier into its component parts; consequently, it becomes necessary to find some other method which may in all cases be generally adopted.

15. The product of any given number may be found by increasing the multiplicand, first, as many times as may be required by the local value, and afterwards as many times as may be required by the simple value of each of the figures in the multiplier.

Thus to multiply 124 by 365; as the first figure 5 in the multiplier has only a simple value, it requires no increase of the multiplicand for its local value.

As the second figure 6 in the multiplier has the local value tens, it requires the multiplicand to be increased ten times for its local value.

And as the third figure 3 in the multiplier has the local value hundreds, it requires the multiplicand to be increased a hundred times for its local value.

Hence by multiplying the multiplicand, and every resulting product by 10, we obtain a series of products corresponding severally to the several local values of the figures in the multiplier.

And if we now multiply each of these products by the simple value of the figure in the corresponding place of the multiplier, we shall obtain a series of products corresponding severally both to the local and simple value of the several figures of the multiplier.

And lastly, if we add together these several products, their sum will be the whole product of the multiplicand multiplied by the whole of the multiplier.

Ex.—To multiply 124 by 365.

$$\begin{array}{l}
124 \times 5 \\
\underline{\quad 10} \\
1240 \times 6 \\
\underline{\quad 10} \\
12400 \\
\underline{\quad\quad 3} \\
37200 = 124 \times 100 \times 3 = 300 \text{ times.} \\
\;\;7440 = 124 \times \;\;10 \times 6 = \;\;60 \text{ times.} \\
\;\;\;\;620 = 124 \times \;\;\;\;5 \;\;\;\;\;= \;\;\;\;5 \text{ times.} \\
\hline
45260 \;\;\;\;\;\;\;\;\;\;\;\;\;\;\;\; = \;\;\;\;\;\;\;\; 365 \text{ times.}
\end{array}$$

16. The Multiplication Table, as far as 12 times 12, should be perfectly committed to memory; but without extending it beyond 9 times 9, the product of any two numbers, consisting of two figures each, may be easily found by mental calculation; for it will always consist of four distinct products.

1st. The product of the tens in the multiplicand multiplied by the tens in the multiplier. 2nd. The product of the units in the multiplicand multiplied by the units in the multiplier. 3rd. Of the tens in the multiplicand multiplied by the units in the multiplier; and 4th. The units in the multiplicand multiplied by the tens in the multiplier.

Ex.—To multiply 98 by 76.

1. 9 tens × 7 tens = 63 hundreds	= 6300		
2. 8 units × 6 units = 48 units	= 48		
		6348	
3. 9 tens × 6 units = 54 tens	= 540		
4. 8 units × 7 tens = 56 tens	= 560		
		1100	
		7448	

Here the first two products, 63 hundreds and 48 units, are added simply by reading them in succession, and give 6348, which is one part of the whole product; and the sum of the second two products, 54 tens and 56 tens, is

instantly seen to be 110 tens or 1100, which is the remaining part of the whole product, and with the same ease may be added to the 63 hundred and 48, making the whole product 7448.

In these calculations it will be better to call the product 63 hundreds, than 6 thousand 3 hundred, as too great a variety of terms may distract the attention.

EXAMPLES FOR PRACTICE.

Multiply	3635276358	by	5
2.	3467607537	×	7
3.	2610005034	×	8
4.	4163763875	×	9
5.	3760807000	×	10
6.	3755736895	×	50
7.	2743705357	×	600
8.	3748765375	×	700
9.	4307536286	×	800000
10.	5763757423	×	9000000
11.	3764537657	×	110
12.	3763745879	×	120

MULTIPLY IN ONE LINE.

13.	3760740000	×	15
14.	5376734763	×	170
15.	7600000587	×	1800
16.	5676747637	×	19000000

17.	3767443874	×	32
18.	7627065386	×	76
19.	7634070837	×	80900
20.	3765763750	×	80009
21.	5000760756	×	90006009
22.	2763752699	×	345
23.	5375236475	×	70809
24.	7076352947	×	800079
25.	3763476352	×	80070090
26.	2346083472	×	567803
27.	1763076502	×	4567807
28.	3657894523	×	70890650
29.	2567090000	×	6708258000

SUBTRACTION.

1. SUBTRACTION is the taking of a smaller number or quantity out of a greater of the same kind, in order to find the difference between them.

Thus, if from 8 units we subtract 3 units, then as $8-3=5$, the remainder, 5, is the difference between the numbers 8 and 3.

2. The remainder or difference between two numbers shows either by how much the greater number exceeds the smaller; or by how much the smaller number is less than the greater.

3. Hence, if the remainder be subtracted from the greater number, it will make it equal to the smaller; or, if the remainder be added to the smaller number, it will make it equal to the greater.

Thus, if $8-3=5$, then $8-5=3$, the smaller number; and $3+5=8$, the greater number:—this is generally regarded as a proof of the correctness of subtraction.

4. The smaller number may always be considered as a part of the greater, for it is actually taken out of it; and as the part must always be of the same kind as the whole, it is evident that the two numbers must be both of the same kind.

5. If the two numbers are not both of the same kind, the subtraction cannot take place, nor the true remainder be found; for the difference between the numbers will not express the difference between the quantities they represent, and consequently cannot be the true remainder.

Thus, in subtracting 3 pence from 8 shillings, if we say $8-3=5$, this remainder, 5, whether considered as 5 shillings or as 5 pence, will not, when subtracted from the greater quantity, 8 shillings, make it equal to the smaller quantity, 3 pence; nor, when added to the smaller quantity, 3 pence, make it equal to the greater quantity, 8 shillings; and consequently cannot be the true remainder or difference between these quantities.

Hence it is evident that in Subtraction, numbers can be subtracted only from others of the same kind.

6. But as there is always a known relation between the several local values in simple Subtraction, and also between the several denominations in compound Subtraction, a certain number of the one always making a unit of the next higher rank, the several parts of the greater number may always be made similar to the corresponding parts of the number to be subtracted, and the subtraction can then take place and the true remainder be found.

Thus, though we cannot subtract 3 units from 8 tens, we may take one of the 8 tens, and, expressing its value in the place of units, consider 8 tens as equal to 7 tens and 10 units; and subtracting the 3 units from the 10 units, we get the remainder, 7 units, which with the 7 tens making 77, is the true remainder; for 8 tens $= 80 - 77 = 3$, the smaller number; and $3 + 77 = 80$, the greater number.

Also, though we cannot subtract 3 pence from 8 shillings, we may take one of the shillings, and expressing its value in the place of pence, consider 8 shillings as equal to 7 shillings and 12 pence; and subtracting the 3 pence from the 12 pence, we get the remainder, 9 pence, which with the 7 shillings making 7 shillings and 9 pence, is the true remainder; for $8s. - 7s.\ 9d. = 3$ pence, the smaller quantity; and $3d. + 7s.\ 9d. = 8$ shillings, the greater.

7. A similar arrangement of the greater number also becomes necessary when any of its parts are smaller than the corresponding parts of the number to be subtracted; for as we cannot take a greater number out of a smaller, the subtraction, without such arrangement, would be impossible.

Thus, in subtracting 28 from 75, as we cannot subtract 8 units from 5 units, we take one of the 7 tens, and expressing its value in the place of units, we consider the number 75 as equal to 6 tens and 15 units, from which, subtracting the 2 tens and the 8 units, we get the remainders, 4 tens and 7 units, together equal to 47, which is the true remainder; for $75 - 47 = 28$, the smaller number, and $28 + 47 = 75$, the greater number.

In this arrangement of the greater number, it is evi-

dent that by increasing the 5 units to 15 units, and diminishing the 7 tens to 6 tens, we make no alteration whatever in the value of the number 75; for $75=60+15$, and $\overline{60+15}-\overline{20+8}=\overline{40+7}=47$, the true remainder.

Here it may be observed, that after having used one of the 7 tens of the upper line, in the place of units, the figure 7, being unaltered in its form, expresses 1 more than its true value; and to counteract this error, a unit is added to the corresponding figure in the lower line before it is subtracted, by which means the true remainder is preserved; for we evidently obtain the same remainder, 4, whether we subtract 2 from 6, or 3 from 7.

This method of preserving the true remainder, is founded on the consideration that the 10 units added in the place of units in the upper line, and the 1 ten added in the place of tens in the lower line, are evidently equal quantities added to each of the numbers 75 and 28, and consequently cannot alter the original difference between them.

Thus, if $75-28=47$, then $\overline{75+10}-\overline{28+10}=47$; for as $10-10=0$, it is evident that the remainder, 47, consists solely of the difference between the original numbers, 75 and 28.

But after having used one of the 7 tens of the upper line, in the place of units, it is quite as easy to diminish the figure 7 to its true value, 6, as to increase the corresponding figure 2 in the lower line to the fictitious value 3; and as this method of diminishing any of the figures in the upper line of which a unit has been used in a lower place, exhibits the real nature of the operation, it is obviously preferable to the artificial method of counteracting one error by the creation of another.

The Rule, therefore, for subtraction, whether of simple numbers or compound quantities, is general, and easily derived from the principle laid down.

RULE.

1st. Place the smaller number under the greater, in such order that its several local values or different denominations may stand directly under those of the greater which are of the same kind.

2nd. Begin at the lowest place or denomination, and subtract each of the figures in the lower line from that figure in the upper line which is of the same kind, writing down under each the remainder, or, if no remainder, a cipher.

3rd. If any of the figures in the upper line be smaller than the figure to be subtracted from it, add to it the value of a unit of the next higher rank, and then subtract; observing always either to diminish by a unit the figure in that place of which a unit has been added in the next lower place, or else to add a unit to the corresponding figure in the lower line before it is subtracted.

4th. Proceed in this manner through all the several local values or different denominations, from the lowest to the highest, writing down under each what remains, or, if nothing remains, a cipher; and these several remainders taken together will be the whole remainder or difference between the given numbers.

Ex.—To subtract 3976 from 5678.

$$\begin{array}{r} 5678 \\ 3976 \\ \hline 1702 \\ \hline \end{array}$$

Here, beginning at the lowest place, we subtract the 6 units in the lower line from the 8 units in the upper line, and get the remainder, 2, which we write under the figure 6 in the place of units. 2. Subtracting the 7 tens in the lower line from the 7 tens in the upper line, we get no remainder; and therefore write a cipher 0 under the figure 7 in the place of tens. 3. As we cannot take 9 hundreds from 6 hundreds, we add to the figure 6 a unit from the 5 in the place of thousands, making 16 hundreds, from which, subtracting the 9 hundreds, we get the remainder, 7, which we write under the figure 9 in the place of hundreds. 4. In subtracting the 3 thousands in the lower line from the 5 thousands in the upper line, we recollect that as 1 of these 5 thousands was used in the place of hundreds, there are only 4 in this place, from which, subtracting the 3 thousands, we get the remainder, 1, which is written under the figure 3 in the place of thousands. 5. These several remainders, taken together, give 1702 for the

whole remainder or difference between the numbers 3976 and 5678; hence $5678-3976=1702$; for $5678-1702=3976$, the smaller number; and $3976+1702=5678$, the greater number.

Ex. 2.—To subtract £7 17*s*. $5\frac{3}{4}$*d*. from £9 15*s*. $6\frac{1}{4}$*d*.

£.	*s*.	*d*.
9 ,,	15 ,,	$6\frac{1}{4}$
7 ,,	17 ,,	$5\frac{3}{4}$
1 ,,	18 ,,	$0\frac{1}{2}$

Here, as we cannot subtract 3 farthings from 1 farthing, we add to the 1 farthing a unit from the 6 in the place of pence, making 5 farthings, from which, subtracting the 3 farthings in the lower line, we get the remainder, 2 farthings, which we write under the line of farthings. 2. In subtracting the 5 pence in the lower line from the 6 pence in the upper, we recollect that as one of the 6 pence was used in the place of farthings, there are only 5 in that place, from which, subtracting the 5 pence in the lower line, we have no remainder, and therefore write a cypher under the line of pence. 3. As we cannot subtract 17*s*. from 15*s*., we add to the 15*s*. a unit from the 9 in the place of pounds, making 35*s*.; from which, subtracting the 17*s*. in the lower line, we get the remainder, 18*s*., which we write under the line of shillings. 4. In subtracting the £7 in the lower line from the £9 in the upper, we recollect that as one of the £9 was used in the place of shillings, there are only 8 in that place, from which, subtracting the £7 in the lower line, we get the remainder, £1, which, with the 18*s*. $0\frac{1}{2}$*d*., gives £1 18*s*. $0\frac{1}{2}$*d*., the whole remainder or difference between the quantities £9 15*s*. $6\frac{1}{4}$*d*. and £7 17*s*. $5\frac{3}{4}$*d*.

EXAMPLES FOR PRACTICE.

From	987656589673	take	123415076321
2.	264316785947	—	15320457
3.	467404321687	—	32527
4.	760412687188	—	630310667057
5.	679407176532	—	357
6.	100410063740	—	5091234576
7.	174611264163	—	76538975
8.	356781000406	—	7614635
9.	261741617684	—	909807
10.	100004600706	—	980

N.B. Other examples may be easily supplied.

DIVISION.

1. Division is the separating of a given number or quantity into any proposed number of equal parts, in order to find the magnitude of each part; or into equal parts of any proposed magnitude, in order to find the number of such parts contained in the whole.

Hence, division is only a more compendious method of subtraction, when the numbers to be subtracted are all equal.

2. In Division, three things are to be considered:—

1. The dividend, which is the number to be divided.
2. The divisor, which shows either the number or the magnitude of the parts into which the dividend is to be divided.
3. The quotient, which shows either the magnitude or the number of the parts contained in the dividend.

3. The divisor and the quotient mutually show the number of each other contained in the dividend, as $24 \div 6 = 4$; here the divisor, 6, shows that there are 6 fours; and the quotient, 4, that there are 4 sixes contained in the dividend, 24; and consequently, that 6 is the fourth part, and 4 the sixth part of 24.

4. Every unit in the quotient is equal to the whole number of units in the divisor; consequently, if the divisor and quotient be multiplied together, their product will be equal to the dividend.

Thus, if $24 \div 6 = 4$; then $6 \times 4 = 24$, the dividend:—this is generally referred to as a test of the correctness of the division.

5. If the divisor be a single unit, 1, the quotient will be equal to the dividend; for it must show the number of units contained in the dividend: thus, $24 \div 1 = 24$.

6. If the divisor be greater than a single unit, as 2, the quotient will be just so many times less than the dividend; for it is evident that there can be only half as many twos as ones contained in the dividend:—thus, $24 \div 2 = 12$.

7. If the divisor be less than a single unit, as a half $\frac{1}{2}$, the quotient will be just so many times greater than the dividend; for it is evident that there must be twice as many halves as wholes of a unit contained in the dividend, thus, $24 \div \frac{1}{2} = 48$.

8. A number may be separated into equal parts of any proposed magnitude, by continually subtracting the divisor from the dividend, either till it is exhausted, or till the remainder becomes less than one of the parts proposed: in this case the number of subtractions will be the quotient; and show how many of those parts are contained in the dividend.

Thus, to divide the number 27 into equal parts, consisting of 6 units each; $27 - 6 = 21$; $21 - 6 = 15$; $15 - 6 = 9$; and $9 - 6 = 3$; here, as the divisor, 6, has been subtracted from the dividend, 27, four times, 4 is the quotient; and shows that there are 4 of these parts contained in the dividend, and 3 units remaining; which remainder is less than one of the parts propsoed.

9. But instead of subtracting the divisor, 6, four several times, as in the preceding example, we may multiply the divisor, 6, by the quotient, 4, and subtract their product from the dividend at once; thus, $6 \times 4 = 24$, and $27 - 24 = 3$, by which we obtain the same result as before, but more expeditiously.

10. We find how many times the divisor may be subtracted from the dividend, by trial; thus, if we suppose that the divisor, 6, may be subtracted from the dividend, 27, five times, then $6 \times 5 = 30$; but 30 cannot be subtracted from 27, therefore the supposed quotient, 5, is too great.

Again, if we suppose that the divisor, 6, can be subtracted from the dividend, 27, only three times, then $6 \times 3 = 18$, and $27 - 18 = 9$; but the divisor, 6, can be again subtracted from this remainder, 9; therefore the supposed quotient, 3, is too small.

Hence we naturally try the intermediate number, 4; and suppose that the divisor, 6, may be subtracted from the dividend, 27, four times; and as $6 \times 4 = 24$, and $27 - 24 = 3$, from which remainder, 3, the divisor, 6, cannot be again subtracted, it is evident that we have found the right quotient, 4.

11. When the dividend consists of many figures, it is separated into several smaller dividends by taking for the first dividend, just as many figures as will contain the divisor, beginning always at the highest place; and the several quotients of these smaller dividends, taken together in the order in which they arise, will be the quotient of the whole dividend.

Thus, to divide the number 2747 by 6; beginning at the highest place, we take for the first dividend 27, which we divide without any minute regard to its particular local value; and finding that the divisor, 6, may be subtracted four times from 27, we write 4 for the first quotient, and subtracting $6 \times 4 = 24$ from this dividend, 27, we get the remainder, 3.

This remainder, 3, whatever may be its particular local value, will justly express 3 tens or 30 in the next lower place, and with the figure 4 in that place of the original dividend will make 34 for the second dividend.

From this second dividend, 34, we find that the divisor, 6, may be subtracted five times; we therefore write 5 for the second quotient; and subtracting $6 \times 5 = 30$ from this second dividend, 34, we get the remainder, 4.

This remainder, 4, whatever may be its particular local value, will be justly reckoned as 4 tens or 40 in the next lower place, and with the figure 7 in that place of the original dividend make 47 for the third dividend.

From this third dividend, 47, we find that the divisor, 6, may be subtracted seven times; we therefore write 7 for the third quotient, and subtracting $6 \times 7 = 42$ from this third dividend, 47, we get the remainder, 5.

We have now divided all the smaller dividends into which the original dividend, 2747, was separated; and have found their respective quotients 4, 5, and 7, which, taken together in that order, give 457 for the whole quotient of the original dividend, 2747, and the remainder, 5.

12. As we always begin at the highest place, both to divide and also to write the quotients, it is evident that the several figures in the quotient will have precisely the same local value as the dividends from which they severally arise.

Thus the figure 4, which has the highest local value, hundreds, is the quotient of the highest dividend, 27 hundreds; the figure 5, which has the local value, tens, is the quotient of the dividend, 34 tens; and the figure 7, which has the lowest local value, units, is the quotient of the lowest dividend, 47 units.

Hence 457 is the quotient of the whole dividend, and shows that there are 457 parts consisting of 6 units each, contained in the dividend 2747, and 5 units remaining.

13. Had this remainder been 6, the divisor would have been contained once more in the dividend, and there would have been one unit more in the quotient; consequently, to express the entire quotient, there must be added to the figures previously found, such part or parts of another unit as the remainder 5 is of the divisor 6.

Now as one 1 is the sixth part of 6, it is evident that 5, which is equal to five ones, must be 5-sixth parts of 6; consequently the entire quotient will be 457, and five-sixths of another unit.

14. These parts of another unit will be properly expressed by writing the divisor underneath the remainder, with a line between them, thus $\frac{5}{6}$; the figure 5 above the line shows that there are 5 parts of the divisor contained in the remainder; and the figure 6 below the line shows that 6 of these parts would be equal to the whole divisor; and consequently that the remainder, 5, is five-sixths of the divisor, and that the entire quotient is $457\frac{5}{6}$.

15. These parts of a unit are called Fractions, of which it may be sufficient at present to understand that a fraction is expressed by two numbers, of which the number above the line is called the Numerator, and shows how many parts of the unit are contained in the fraction; and the number below the line is called the Denominator, and shows how many of these parts are contained in the whole unit.

Thus, if a unit, for instance an apple, be divided into

eight equal parts, and we take one of these parts, we shall have one-eighth of the apple, or $\frac{1}{8}$; if we take two of these parts we shall have two-eighths of the apple, or $\frac{2}{8}$; if three, $\frac{3}{8}$; if four, $\frac{4}{8}$; and so on; and if we take eight of these parts, we shall have all the parts into which the apple was divided, and consequently $\frac{8}{8}=$ the whole.

Here we may observe, that in the fraction $\frac{2}{8}$, the numerator 2 is just the fourth part of the denominator 8, or that the number of parts in the fraction is just one-fourth of the number of parts in the whole unit; consequently the fraction $\frac{2}{8}$ is equal to the fraction $\frac{1}{4}$; and in the fraction $\frac{4}{8}$, the numerator 4 is just one-half of the denominator 8; consequently the fraction $\frac{4}{8}=\frac{1}{2}$.

Any further inquiry into the theory of fractions at present, would be only an unprofitable anticipation of a subject which belongs more properly to that part of Arithmetic which we regard as a science.

16. When the divisor is a single figure, or any number within the limits of the Multiplication Table, the multiplication of the divisor by the quotient, and the subtraction of their product from the dividend are performed mentally, and the quotients only are written down; and in this case, always underneath their respective dividends, as in the annexed example. This is called Short Division.

$$\begin{array}{r} 6)2747 \\ \hline 457\frac{5}{6} \\ \hline \end{array}$$

17. When the divisor consists of several figures, the whole of the operation is written down at length; and the quotients in this case are placed on the right hand of the dividend, as in the annexed example. This is called Long Division.

$$\begin{array}{l} 25)2747(109\frac{22}{25} \\ \quad\;\; 25 \\ \quad\;\; \overline{\;\;247} \\ \quad\;\;\;\; 225 \\ \quad\;\;\;\; \overline{\;\;22} \\ \end{array}$$

Rule.

1. Place the divisor on the left hand of the dividend, and find how many times it may be subtracted from as many of the highest figures of the dividend as are sufficient to contain it, and place the figure expressing the quotient on the right hand of the dividend.

2. Multiply the divisor by the figure placed in the

quotient, and write down the product under that part of the dividend in which it is contained, always regarding the lowest place in every dividend as the place of units.

3. Subtract this product from the dividend under which it is written, and to the right hand of the remainder bring down the figure in the next lower place of the dividend, for a new dividend, with which proceed as before.

4. If the divisor be not contained in this new dividend, write a cipher in the quotient, and place the next lower figure of the dividend on the right hand of the former, for a new dividend, with which proceed as before.

5. Continue this process till all the figures of the original dividend have been brought down; taking care always to place either a figure or a cipher in the quotient, for every figure brought down from the dividend; and the several figures in the quotient taken together will be the quotient of the whole dividend.

6. If after the whole of the dividend has been divided, there be any remainder, annex to the quotient already found, such part or parts of another unit, as this remainder is of the divisor.

18. The divisor is sometimes the product of two or more single figures; as 24 is the product of $4 \times 6 = 24$; and 192 the product of $4 \times 6 \times 8 = 192$.

In this case, instead of dividing by the whole divisor at once as in Long Division, the dividend may be divided by any one of these single figures, as in Short Division; the quotient thus found by another of them, and every succeeding quotient by another, till all have been used; and the last quotient will be the quotient of the original dividend divided by the whole divisor.

Ex.—To divide 5678 by $24 = 4 \times 6$.

Here first dividing the dividend 5678 by 4, we get the quotient 1419, which is four times less than the original dividend 5678; and now dividing this quotient 1419 by 6, we get the second quotient 236, which is six

```
4)5678
  ----
6)1419..2
  ----
   236..3
  ----
```

times less than the first quotient 1419, or twenty-four times less than the original dividend 5678, and is consequently the quotient of the original dividend divided by the whole divisor 24.

19. By this method of resolving the divisor into its component parts, and dividing by each of them separately, we obtain the two several remainders 2 and 3, from which we may easily determine the true remainder, by the simple consideration that every unit in every remainder must have precisely the same value as the units in the dividend of which it is a part.

Now in the example before us, as the first dividend, 5678, consists of units, the first remainder, 2, which is a part of it, must be 2 units; and as the second dividend, 1419, is the quotient of the first dividend divided by 4, and consequently consists of fours of units, for it shows the number of fours contained in 5678; so the second remainder, 3, which is part of this second dividend, must be 3 fours of units; and the true remainder will be 2 units $+3$ fours, or $2+12=14$.

Here it may be observed that the value of a unit in the second remainder is 4 times as great as the value of a unit in the first remainder; or just as many times as the divisor 4 is greater than a single unit; consequently if the second remainder, 3, be multiplied by the first divisor, 4, we shall have $3\times4=12$ units of the same value as those of the first remainder, 2; and the sum of these $12+2=14$ will be the true remainder.

Hence, whatever may be the number of component parts into which the divisor may be resolved, we have the following general rule for finding the aggregate value of the several remainders, and consequently the true remainder.

20. Multiply the last remainder by the last divisor but one, and to their product add the next preceding remainder; multiply the sum thus found, by the next preceding divisor, and to their product add the next preceding remainder, and continue this process till you have multiplied by the first divisor and added the first remainder; this last sum will be the true remainder.

Ex.—To divide 5678 by $192=4\times6\times8=192$.

4)5678
6)1419..2=2 units = 2
8) 236..3=3 fours = 12
29..4=4 sixes of fours=96
110

192)5678(29$\frac{110}{192}$
384
1838
1728
110 remdr.

Here $4\times6=24$; $24+3=27$; and $27\times4=108$; $108+2=110$.

21. We have seen (NUMERATION, Art. 15), that a number is decreased ten times for every cipher cut off from the right hand of it; consequently, to divide a number by a divisor having only a local value, as 10, 100, 1000, &c., we have only to cut off from the right hand of it as many places as there may be ciphers in the divisor.

If the places cut off from the right hand of the number consist of ciphers, the figures on the left hand will be the exact quotient without any remainder; thus $123000\div1000=123$ exactly; the three ciphers cut off, ,000 being equal to nothing.

If the places cut off consist of digits, these digits will form the remainder, and must be annexed to the quotient as so many parts of another unit; thus $123456\div1000=123$, and the three digits cut off, ,456, will be the remainder, which, as the divisor is 1000, are 456 thousandth parts of the divisor, and the entire quotient is $123\frac{456}{1000}$.

EXAMPLES FOR PRACTICE.

In Short Division.

1. Divide 6781 by 2
2. $4668796321 \div 3$
3. $3716791642 \div 4$
4. $6712314763 \div 5$
5. $3310714671 \div 6$
6. $7163742834 \div 7$
7. $5741000730 \div 8$
8. $7160355675 \div 9$
9. 467046980 by 10
10. $658932469 \div 11$
11. $376453902 \div 12$
12. $479846239 \div 100$
13. $978536469 \div 1000$
14. $407068956 \div 50000$
15. $376543958 \div 700000$
16. $576098460 \div 9000000$

By the Component Parts of the Divisor.

17. 3456789416 by 16
18. 2016347153 ÷ 25
19. 6216101071 ÷ 48
20. 1010106059 ÷ 64
21. 2160412160 ÷ 720
22. 1216012516 ÷ 8100
23. 45678946 by 125
24. 716043760 ÷ 216
25. 260083706 ÷ 432
26. 612163470 ÷ 5040
27. 612345687 ÷ 72900
28. 612341600 ÷ 89600

In Long Division.

29. 4568787561 by 23
30. 4160001762 ÷ 34
31. 7160912656 ÷ 57
32. 4176261352 ÷ 68
33. 6123460012 ÷ 92
34. 7163572761 ÷ 167
35. 6124162134 ÷ 365
36. 741236471 by 563
37. 345678213 ÷ 789
38. 463478123 ÷ 8960
39. 416327653 ÷ 98775
40. 175987495 ÷ 87650
41. 212600746 ÷ 99875
42. 680943675 ÷ 90807

COMPOUND ADDITION.

1. Compound Addition is the collecting of several quantities of the same kind, but of different denominations, into one sum; and, in principle, differs in no respect from Simple Addition, which has been already explained. (See Addition.)

2. The only difference in practice is, that in Simple Addition the number of units in every lower denomination which make one of the next higher, is always uniform; as, for instance, 10 units make 1 ten; 10 tens 1 hundred, &c.

Whereas, in Compound Addition, the number of units in any lower denomination which make a unit of the next higher, varies according to the nature of the quantities to be added; as, in money, 4 farthings make 1 penny, 12 pence 1 shilling, &c.

Hence, we have only in Compound Addition to find how many units of the next higher denomination are contained in the sum of every lower denomination, and in all other respects to proceed exactly as in Simple Addition.

EXAMPLES FOR PRACTICE.

1. Add £36 17*s*. 6*d*. + £1 17*s*. 6½*d*. + 17*s*. 11½*d*. + 2½*d*. + £1 0*s*. 6¼*d*. + £67 13*s*. 6½*d*. + £19 10*s*. 11½*d*. + 19*s*. 11¾*d*.

2. Add £167 10*s*. 0¼*d*. + £37 19*s*. 10½*d*. + £21 16*s*. + £176 4*s*. 6½*d*. + £36 + 11½*d*. + £371 0*s*. 6½*d*. + £10 + £174 13*s*. 4*d*.

3. Add £1267 13*s*. 6½*d*. + £31 17*s*. 11*d*. + £3 17*s*. 1½*d*. + £21 16*s*. 3½*d*. + 5*s*. + £36 + 1*d*. + £306 10*s*. + £27 13*s*. 3¾*d*.

4. Add £3764 16*s*. 9½*d*. + £12 17*s*. 9¼*d*. + £261 19*s*. 11*d*. + £21 0*s*. 1½*d*. + £129 6*s*. 3½*d*. + £26 10*s*. 2*d*. + £16 1*s*. 4*d*. + £231 15*s*. 6½*d*.

5. Add £3624 12*s*. 6*d*. + £111 19*s*. 11*d*. + £21 16*s*. 3*d*.

+£1674 19*s.* 7½*d.*+£126 19*s.* 9½*d.* + £16 19*s.* 8½*d.*+ £117 13*s.* 5½*d.*+£36 19*s.* 11½*d.*

6. £364 15*s.* 6*d.*+£117 2*s.* 6½*d.* + £129 16*s.* 5½*d.* + £298 17*s.* 3¼*d.*+£57 19*s.* 7½*d.*+£57 15*s.* 11¼*d.*+£23 16*s.* 11½*d.*+£762 16*s.* 11¾*d.*

WEIGHTS AND MEASURES.

Troy Weight.

1. Add 12 lb. 9 oz. 13 dwts. 12 gr. + 11 oz. 16 dwts. 13 grs.+16 dwts. 19 grs.+1 lb. 6 oz. 17 dwts. 14 grs. + 24 lb. 3 oz. 0 dwts. 16 grs.+16 dwts. 15 grs.+19 grs.

2. Add 16 oz. 14 dwts. 10 grs.+21 lb. 4 oz. 10 dwts. + 26 lb. 9 oz. 4 dwts. 13 grs. + 5 oz. 8 dwts. 9 grs. + 16 dwts. 13 grs.+126 lb. 6 oz. 14 dwts. 11 grs.

Avoirdupois Weight.

3. Add 36 ton. 14 cwt. 3 qrs. + 15 cwt. 2 qrs. 18 lb.+ 3 qrs. 17 lb. 13 oz.+19 lb. 12 oz. 15 drs.+15 ton. 12 cwt. 3 qrs. 12 lb. 9 oz.+3 qrs. 19 lb. 12 oz. 16 drs.

4. Add 16 lb. 12 oz. 14 drs.+7 lb. 13 oz. 9 drs.+3 qrs. 17 lb. 5 oz. 12 drs. + 40 ton. 19 cwt. 27 lb. 9 drs. + 13 cwt. 2 qrs. 18 lb. 11 oz. 14 drs. + 3 qrs. 26 lb. 9 oz. 7 drs.

Apothecaries' Weight.

5. Add 7 lb. 9 ℥ 5 ʒ 2 ℈ +5 ℥ 6 ʒ 1 ℈ +4 ʒ 2 ℈ 17 grs. + 1 lb. 10 ℥ 7 ʒ 2 ℈ 14 grs. + 11 ℥ 3 ʒ 1 ℈ 13 grs. + 1 ℈ 9 grs.

6. Add 11 ℥ 5 ʒ 1 ℈ 12 grs. + 12 lb. 9 ℥ + 2 ℈ 15 grs. + 7 ʒ 2 ℈ 12 grs. + 12 lb. 3 ℥ 5 ℈ 18 grs. + 5 ℥ 2 ʒ 19 grs.

Cloth Measure.

7. Add 27 yds. 3 qrs. 2 n. + 16 yds. 2 qrs. 3 n. + 1 yd. 3 qrs. 3 n. + 67 yds. 1 qr. 2 n. + 3 qrs. 2 n. + 17 yds. 3 qrs. 2 n.

8. Add 27 E. ells. 4 qrs. 3 n. + 19 E. ells. 2 qrs. 3 n.+ 16 E. ells. 3 qrs. 2½ n. + 3 E. ells. 2 qrs. 3½ n.

9. Add 23 Fl. ells. 2 qrs. 2 n. + 16 Fl. ells. 2 qrs. 3 n. +15 Fl. ells. 2 qrs. 3½ n.+2 qrs. 2½ n.+25 Fl. ells. 1 qr. 2 n. + 12 Fl. ells. 2 qrs. 3 n.

10. Add 21 Fr. ells. 5 qrs. 3 n. + 16 Fr. ells. 4 qrs. 2½ n. + 16 Fr. ells. 3 qrs. 2½ n. + 5 qrs. 3 n. + 1½ n.

Long Measure.

11. Add 16 ft. 11 in. 2 bar. + 9 ft. 10 in. 2 bar. + 6 ft. 11 in. 1 bar. + 3 ft. 7 in. 2 bar. + 12 ft. 8 in. 1 bar. + 13 ft. 9 in. 2 bar.

12. 27 yds. 2 ft. 11 in. 2 bar. + 10 yds. 2 ft. 10 in. 2 b. + 29 yds. 2 ft. 2 bar. + 19 yds. 1 ft. 1 bar. + 13 yds. 2 ft. 11 in. 2 bar. + 9 in. 2 bar.

13. Add 20 m. 7 fur. 39 p. + 16 m. 6 fur. 26 p. 3½ yds. + 5 fur. 28 p. + 4½ yds. + 15 m. 5 fur. 17 p. 3 yd. 2 ft. 11 in. + 1 yd. 2 ft. 8 in. 2 bar.

Wine Measure.

14. Add 21 hhd. 40 gal. 3 qt. + 21 gal. 2 qt. 1 pt. + 15 hhd. 16 gal. 3 qt. 1 pt. + 19 hhd. 24 gal. 3 qt. + 17 gal. 2 qt. 1½ pt. + 3 qt. 1½ pt.

15. Add 26 p. 1 hhd. 17 gal. + 17 p. 1 hhd. 19 gal. 2 qt. + 17 gal. 3 qt. 1 pt. + 1 hhd. 19 gal. 3 qt. 1 pt. + 2 qt. 1½ pt. + 27 gal. 2 qt. 1½ pt.

Ale and Beer Measure.

16. Add 17 bar. 1 firk. 8 gal. + 6 bar. 2 fir. 8 gal. 2 qt. + 10 bar. 3 firk. 7 gal. + 12 bar. 3 firk. 8 gal. 3 qt. + 21 bar. 2 firk. 6 gal. 2 qt.

17. Add 20 hhd. 1 bar. 3 firk. 5 gal. + 1 hhd. 1 bar. 3 firk. 7 gal. + 27 hhd. 1 bar. 2 firk. 8 gal. + 7 gal. 2 qt. 1 pt. + 10 bar. 3 firk. 4 gal. 1 qt.

Dry Measure.

18. Add 36 bu. 3 pec. 1 gal. 3 qts. + 15 bu. 2 pec. 1 gal. 3 qt. + 16 bu. 2 pec. 1 gal. 2 qt. + 1 gal. 3 qts. 1 pt. + 25 bu. 1 pec. 1 gal. 3 qt. 1½ pt. + 2 bu. 3 pec. 1 gal. 3 qt. 1½ pt.

19. Add 26 qr. 7 bu. 2 pec. + 5 bu. 2 pec. 1 gal. 3 qt. + 16 qr. 5 bu. 3 pec. 3 qt. + 4 bu. 2 pec. 1 gal. 3 qt. 1½ pt. + 25 qr. 4 bu. 2 pec. + 6 bu. 3 pec. 1 gal. 3 qt. 1½ pt. + 2 bu. 3 pec. 1 gal. 3 qt. 1 pt.

Time.

20. Add 36 d. 16 h. 24 m. + 18 d. 14 h. 54 m. 16 sec. + 1 d. 20 h. 25 m. 37 sec. + 115 d. 17 h. 28 m. 27 sec. + 134 d. 18 h. 19 m. 49 sec. + 3 h. 57 m. 49 sec.

COMPOUND MULTIPLICATION.

1. Compound Multiplication is the repeating of a given quantity consisting of several different denominations any proposed number of times, and finding its amount when so many times repeated.

2. Compound Multiplication differs from Simple Multiplication chiefly in the different relations which a unit in any lower denomination may bear to a unit of the next higher, which, instead of being constantly a relation of ten to one, as in Simple Multiplication, varies according to the nature of the quantity to be multiplied.

3. Hence it becomes necessary, in every different example, to find how many units of the next higher denomination are contained in the product of every lower denomination; and for the same reason, when the multiplier consists of several figures, the several products cannot be increased by position for the local value of the multiplier, as in Simple Multiplication.

4. The method, therefore, of resolving the multiplier, when it consists of more than one figure, into its several component parts, and multiplying successively by each part; and also the method of increasing the multiplicand as many times as may be required both for the local and simple values of the several figures in the multiplier, become necessary in the multiplication of compound quantities.

But as both these methods have been fully explained in Simple Multiplication, it will here be sufficient to give only a detailed example of each. (See Multiplication.)

Ex. 1.—To multiply £12 13*s*. 8¼*d*. by 9.

Here the product of the farthings is 9, which contains 2 pence and 1 farthing, we therefore write ¼ under the line of farthings.

2. The product of the pence is 72, to which we add the 2 pence contained in the product of farthings, making 74 pence, which contains

£.	*s*.	*d*.
12 „	13 „	8¼
		9
£114 „	3 „	2¼

6 shillings and 2 pence; we therefore write 2 in the place of pence. 3. The product of the shillings is 117, to which we add the 6 shillings contained in the product of the pence, making 123 shillings, which contain £6 and 3 shillings; we therefore write 6 in the place of shillings. 4. The product of the pounds is 108, to which we add the £6 contained in the product of the shillings, making £114, the whole of which we write in the place of pounds; and these several products give the whole product, £114 3*s.* 2¼*d.*

Ex.—2. To multiply £1 15*s.* 7*d.* by $192 = 4 \times 6 \times 8$.

£.	*s.*	*d.*	
1	15	7	
		4	
7	2	4	$= 4$ times.
		6	
42	14	0	$= 4 \times 6 = 24$ times.
		8	
£341	12	0	$= 4 \times 6 \times 8 = 192$ times.

Here, dividing the multiplier, 192, into the component parts, $4 \times 6 \times 8$, we first multiply the multiplicand, £1 15*s.* 7*d.*, by one of these parts, 4, and obtain the product, £7 2*s.* 4*d.*, which is 4 times as great as the multiplicand, £1 15*s.* 7*d.*; we next multiply this product by another of these component parts, 6, and obtain the product, £42 14*s.* 0*d.*, which is 6 times as great as the product, £7 2*s.* 4*d*, or $4 \times 6 = 24$ times as great as the original multiplicand, £1 15*s.* 7*d.*; and lastly, multiplying this product by the last of the component parts, 8, we obtain the product, £341 12*s.* 0*d.*, which is 8 times as great as the product, £42 14*s.* 0*d.*, or $4 \times 6 \times 8 = 192$ times as great as the original multiplicand, £1 15*s.* 7*d.*, and consequently the whole product of the given quantity multiplied by the whole multiplier, 192.

Ex. 3.—To multiply £1 6*s*. 5*d*. by 365.

£1 6*s*. 5*d*. × 5,			
10			
13 „ 4 „ 2 × 6,			
10			
132 „ 1 „ 8			
3			
396 „ 5 „ 0	=£1 6*s*. 5*d*. × 100 × 3	= 300	times.
79 „ 5 „ 0	=£1 6*s*. 5*d*. × 10 × 6	= 60	times.
6„12 „ 1	=£1 6*s*. 5*d*. × 5	= 5	times.
£482 „ 2 „ 1	=	365	times.

Here, multiplying the given quantity by 10, we get the product, £13 4*s*. 2*d*., which is 10 times as great as the multiplicand, £1 6*s*. 5*d*., and is the increase required by the local value of the figure 6 in the multiplier, 365. 2. Multiplying this product, £13 4*s*. 2*d*., by 10, we get the product £132 1*s*. 8*d*., which is 10 times as great as the product, £13 4*s*. 2*d*., or 10×10=100 times as great as the multiplicand, £1 6*s*. 5*d*., which is the increase required by the local value of the figure 3 in the multiplier, 365; and now multiplying this last product, £132 1*s*. 8*d*., by 3, we get the product, £396 5*s*. 0*d*., which is 3 times as great as the product, £132 1*s*. 8*d*., or 100 × 3=300 times as great as the original multiplicand, £1 6*s*. 5*d*., which is the increase required both by the local and simple values of the figure, 300. In the same manner, multiplying the product, £13 4*s*. 2*d*., by 6, we get the product, £79 5*s*., which is 6 times as great as the product, £13 4*s*. 2*d*., or 10×6=60 times as great as the original multiplicand, £1 5*s*. 6*d*., and is the increase required both by the local and simple values of the figure, 60; and lastly, multiplying the original multiplicand, £1 6*s*. 5*d*., by 5, we get the product, £6 12*s*. 1*d*., 5 times as great as the given quantity, £1 6*s*. 5*d*., which is the whole increase required by the simple value of the figure 5, which has no local value; and adding together these three last products, we get their sum, £482 2*s*. 1*d*., which is the whole product of the multiplicand, £1 6*s*. 5*d*., multiplied by the whole multiplier, 365.

EXAMPLES FOR PRACTICE.

In One Line.

	£.	s.	d.			£.	s.	d.
1.	21675	16	11¼ × 2		6.	31617	15	11½ × 8
2.	31789	7	0½ × 3		7.	27165	14	3¾ × 9
3.	37061	13	4¾ × 4		8.	76173	18	10¼ × 10
4.	50760	19	9½ × 5		9.	25276	12	9½ × 11
5.	71685	17	7¼ × 6		10.	32099	13	8¾ × 12

By the Component Parts.

	£.	s.	d.			£.	s.	d.
11.	156	17	6 × 15		19.	287	17	9½ × 216¼
12.	376	13	4½ × 16		20.	796	13	4 × 336½
13.	174	16	8¾ × 24½		21.	164	13	9½ × 176
14.	216	19	7¼ × 36		22.	375	12	11¼ × 365½
15.	178	11	11 × 48¼		23.	216	11	7½ × 971
16.	576	12	9½ × 54		24.	406	10	10½ × 1076¾
17.	275	17	6¾ × 63		25.	311	11	8¼ × 3116
18.	164	16	3⅜ × 72		26.	176	13	9½ × 5417½

WEIGHTS AND MEASURES.

1. 79 lb. 4 oz. 16 dwts. 12 grs. × 9½
2. 37 ton. 15 cwt. 3 qrs. 26 lbs. × 7
3. 1 ton. 12 cwt. 2 qrs. 25 lbs. 14 oz. × 12
4. 15 cwt. 2 qrs. 17 lb. 14 oz. 12 dr. × 11½
5. 33 lb. 9 ℥ 5 ʒ 2 ℈ × 12
6. 16 lb. 7 ℥ 7 ʒ 1 ℈ × 15
7. 27 yds. 2 qrs. 2½ n. × 24
8. 37 E. ells. 4 qrs. 3 n. × 36
9. 174 Fl. ells. 2 qrs. 3 n. × 48
10. 263 Fr. ells. 5 qrs. 3 n. × 96
11. 37 p. 3 yds. 2 ft. 9 in. 2 bar. × 132½
12. 27 mi. 6 fur. 35 p. 4½ yds. × 125
13. 23 hhds. 61 gal. 3 qt. 1 pt. × 63
14. 12 p. 1 hhd. 45 gal. 3 qt. 1 pt. × 72
15. 17 bar. 3 fir. 7 gal. 3 qt. × 42½
16. 36 bar. 2 fir. 5 gal. 2 qt. 1 pt. × 150
17. 25 bu. 3 pu. 1 gal. 3 qt. × 81½
18. 36 qurs. 7 bu. 3 pu. 1 gal. 3 qt. × 216
19. 365 da. 6 ho. 48 min. 58 sec. × 54

COMPOUND SUBTRACTION.

1. Compound Subtraction is the taking of a smaller quantity consisting of several denominations out of a greater quantity of the same kind also consisting of several denominations, in order to find the difference between them.

2. Compound Subtraction differs from Simple Subtraction only in the different relations which a unit in any of the lower denominations may bear to a unit of the next higher denomination, according to the nature of the quantities whose difference is to be found.

3. We have, therefore, when any quantity in the upper line is smaller than the corresponding quantity to be subtracted from it, only to find how many units must be added to it for a unit of the next higher denomination, and proceed exactly as in Simple Subtraction.

EXAMPLES FOR PRACTICE.

	£.	s.	d.		£.	s.	d.
1.	21098117	16	6½	—	376461	15	7
2.	46921367	10	4½	—	764761	15	10½
3.	43076564	14	3½	—	337276	0	5¼
4.	75212762	13	4¼	—	101717	19	2
5.	36090762	4	7¼	—	341067	15	5½
6.	47657695	8	6½	—	164296	13	5½
7.	21340374	15	7¼	—	223419	11	6½
8.	20104127	0	0¼	—	136469	17	8¾
9.	67120700	10	6½	—	373079	19	7¼
10.	31506764	15	3¼	—	475920	10	9¼
11.	54203367	0	6	—	116906	18	0¾
12.	16924764	1	8	—	366197	9	11½

WEIGHTS AND MEASURES.

1. 24 lb. 9 oz. 15 dwts. 14 qrs. — 11 oz. 16 dwts. 17 grs.
2. 16 lb. 4 oz. 10 dwts. 9 oz. 18 dwts. 19 grs.
3. 37 cwt. 2 qrs. 21 lb. — 15 cwt. 3 qrs. 27 lb. 13 oz.
4. 150 ton. 21 lb. — 17 cwt. 1 qr. 26 lb. 12 oz. 13 dr.
5. 32 lb. 9 ℥ 5 ʒ — 16 lb. 11 ℥ 6 ʒ 2 ℈.
6. 61 lb. 3 ℥ 4 ʒ — 28 lb. 9 ℥ 7 ʒ 1 ℈.
7. 27 yds. 1 qr. 2 n. — 16 yds. 3 qrs. 3 n.
8. 116 E. ells. 2 qrs. 1 n. — 110 E. ells. 4 qrs. 2½ n.
9. 21 Fr. ells. 4 qrs. 2 n. — 16 Fr. ells. 5 qrs. 3½ n.
10. 61 fur. 31 pol. 3 yds. — 28 fur. 37 po. 5 yds. 2 ft.
11. 27 m. 3 fur. 14 po. — 37 po. 4 yds. 2 ft. 9 in. 1 bar.
12. 23 hhds. 41 gals. 2 qts. — 15 hhds. 52 gal. 3 qts. 1 pt.
13. 12 hhds. 31 gal. 1 qt. — 3 hhds. 47 gal. 2 qts. 1½ pt.
14. 17 bar. 1 firk. 3 gal. — 15 bar. 1 firk. 7 gal. 3 qts.
15. 67 bar. 2 firk. 2 gal. — 26 bar. 2 firk. 6 gal. 2 qts.
16. 13 bu. 1 pec. 1 gal. — 6 bu. 3 pec. 1 gal. 2 qts.
17. 14 qrs. 6 bu. 2 pec. — 18 qrs. 7 bu. 3 pec. 1 gal.
18. 274 d. 16 h. 17. m. — 168 d. 18 h. 47 m.
19. 163 d. — 27 d. 17 h. 19 m. 27 sec.

COMPOUND DIVISION.

1. Compound Division is the separating of a given quantity, consisting of several denominations, into any proposed number of equal parts, in order to find the magnitude of each part.

2. In Compound Division there will be always as many separate dividends as there may be different denominations in the quantity to be divided, each of which is divided exactly as in Simple Division; hence Compound Division is only a repetition of the same process as in Simple Division.

3. There is, however, this difference, that we cannot, as in Simple Division, by placing on the right hand of the last remainder, as many of the next lower figures as may be requisite, form a new dividend in which that remainder will by position justly express its true local value.

4. We must, therefore, make the last remainder from every higher dividend as many times greater by Multiplication as a unit in the next lower denomination is less than a unit in the dividend of which it is a part; and then adding to it the number in that denomination, we shall have the next lower dividend, with which we proceed exactly as in Simple Division. (See Division.)

Ex.—To divide £1357 16*s*. 11½*d*. by 37.

Here, dividing the highest dividend, £1357 exactly as in Simple Division, we get the quotient, £36, and the remainder, £25; and making this remainder as many times greater as a unit in the next lower denomination, shillings, is less than a pound, by multiplying it by 20, we add to the product, 16*s*., the number in the next lower denomination, and get the next lower dividend, 516*s*. 2. Dividing this dividend in the same manner, we get the quotient, 13*s*., and the remainder, 35*s*.; and multiplying this remainder

```
      £.   s.  d.     £.  s.  d.
37) 1357  16  11½ ( 36  13  11½ 24/37
    111
    ---
     247
     222
     ---
      25
      20
      ---
 37) 516 ( 13s.
     37
     ---
     146
     111
     ---
      35
      12
      ---
  37) 431 ( 11d.
      37
      ---
       61
       37
       --
       24
        4
       --
   37) 98 ( 2
       74
       --
       24
       --
       37
```

by 12, and adding to the product, 11, the number in the next lower denomination, pence, we get the next lower dividend, 431*d.*, which we divide as before, and get the quotient, 11*d.*, and the remainder, 24*d.*, which we multiply by 4; and adding to the product, 2, the number in the next lower denomination, farthings, get the next lower dividend, 98 farthings, which, dividing as before, we get the quotient, 2 farthings, and the remainder, 24; which, as there is no lower denomination in the given quantity, we annex to the quotient as $\frac{24}{37}$ of another farthing. Lastly, taking all these several quotients together, we get the whole quotient of the whole dividend, £36 13*s.* 11½*d.* $\frac{24}{37}$.

EXAMPLES FOR PRACTICE.

	£.	s.	d.	
1.	326204516	17	6	÷ 2
2.	217673764	16	11½	÷ 3
3.	420534760	0	7½	÷ 4
4.	410636140	17	11½	÷ 5
5.	320136140	0	7¾	÷ 6
6.	469027604	18	11½	÷ 7
7.	609836785	17	5	÷ 8

	£.	s.	d.	
8.	402612671	17	3¼	÷ 9
9.	504061674	16	5¾	÷ 10
10.	420141764	16	5⅓	÷ 11
11.	640244167	13	5	÷ 12

By the Component Parts of the Divisor.

	£.	s.	d.	
12.	509146757	17	6½	÷ 16
13.	750146174	0	6¾	÷ 36
14.	426023476	4	0	÷ 63
15.	430261517	16	11¼	÷ 125
16.	990446176	13	8¼	÷ 216
17.	543266475	10	10	÷ 504
18.	670596476	15	4	÷ 729

In Long Division.

	£.	s.	d.	
19.	16746	17	6	÷ 23
20.	6466471	15	6¾	÷ 98
21.	2674763	16	7¼	÷ 163
22.	451780645	16	11½	÷ 365
23.	3056745416	16	5¼	÷ 7897
24.	21914760953	15	1½	÷ 10626
25.	74214164741	13	2	÷ 781365
26.	50726164716	17	6¾	÷ 8943254
27.	32131126471	15	3¼	÷ 97894076

WEIGHTS AND MEASURES.

1. 3164 lb. 11 oz. 17 dwts. 18 grs. ÷ 11
2. 3712 cwt. 3 qrs. 17 lbs. 14 oz. ÷ 12
3. 127 lb. 9 ℥ 7 ʒ 2 ℈ ÷ 42
4. 316 lb. 11 ℥ 6 ʒ 1 ℈ ÷ 63
5. 3276 yds. 3 qrs. 2 n. ÷ 72
6. 3767 E. ells. 2 qrs. 3 n. ÷ 16.
7. 1675 Fl. ells. 1 qr. 2 n. ÷ 64
8. 347 mi. 7 fur. 38 po. 4 yds. 2 ft. ÷ 125
9. 734 po. 4 yds. 2 ft. 9 in. 2 bar. ÷ 216
10. 371 hhds. 49 gal. 3 qts. 1 pt. ÷ 99
11. 2716 chal. 28 bu. 2 pec. ÷ 729
12. 3764 d. 17 h. 37 m. ÷ 63
13. 1276 h. 24 m. 57 sec. ÷ 125

REDUCTION.

1. Reduction is the changing of a given number or quantity of any one name or denomination, into an equivalent number or quantity of any other name or denomination required.

2. In reducing quantities of one denomination into equivalent quantities of another, it is essential that a certain number of units of the one, be equal to a unit of the other denomination; otherwise the reduction cannot take place.

Thus, we may reduce any number of guineas into an equivalent number of shillings, because 21 shillings are equal to 1 guinea; but we cannot reduce bushels into miles; for it is evident that no number of miles can be equal to a bushel, nor any number of bushels be equal to a mile.

3. A rule for Reduction may easily be deduced from the obvious principle, that the original value of the given quantity must be always the same in all the various denominations into which it may be reduced.

Now, it is evident that the same number cannot retain the same value under two different denominations, in one of which the value of a unit may be greater or less than the value of a unit in the other.

4. Consequently whenever the name of the given quantity is to be changed, it is evident that the number representing it must also be changed.

5. If a quantity be reduced from a higher name into a lower, its original value will, by the change of the name, be decreased as many times as the lower name is less valuable than the higher; consequently, to counteract this decrease, the given number must be made as many times greater as the name is made less valuable.

6. If a quantity be reduced from a lower name into a

higher, its original value will, by the change of the name, be increased as many times as the higher name is more valuable than the lower; consequently, to counteract this increase, the given number must be made as many times less as the name is made more valuable.

7. It is obvious that in the one case we obtain a greater number of units, each of which is just so many times less valuable; and in the other case, a smaller number of units, each of which is just so many times more valuable than a unit of the given quantity; and, consequently, in both denominations, the original value of the given quantity is the same.

Ex.—To reduce 5 crowns into an equivalent number of shillings.

Here, as a unit of the required name, a shilling, is five times less valuable than a unit of the given name, we make the given number 5 five times greater, and obtain $5 \times 5 = 25$, the equivalent number of shillings.

Ex. 2.—To reduce 25 shillings into an equivalent number of crowns.

Here, as a unit of the required name, crowns, is five times more valuable than a unit of the given name, we make the given number 25 five times less, and obtain $25 \div 5 = 5$, the equivalent number of crowns.

RULE.

1st. Make the given number as many times greater as the value of a unit of the required name is less, or as many times less as the value of a unit of the required name is greater, than the value of a unit of the given name.

2nd. If the given quantity consist of several denominations, begin with the highest, and reduce the number of that denomination into the next lower, adding to the product, the number of that name which is found in the given quantity.

3rd. Proceed in this manner through all the several denominations from the highest to the lowest, adding always to every inferior denomination the number of that name in the given quantity.

Ex.—To reduce £37 „ 15*s*. „ 6*d*. into pence.

Here we first reduce the £37 into shillings; and as a shilling is 20 times less valuable than a pound, we make the given number, 37, 20 times as great, and obtain $37 \times 20 = 740$, the equivalent number of shillings; to which we add the 15 shillings of the given quantity, and obtain $740 + 15 = 755$, the whole number of shillings in the given quantity. We next reduce these 755 shillings into pence; and as a penny is 12 times less valuable than a shilling, we make the number, 755, 12 times as great, and obtain 9060, the equivalent number of pence; to which we add the 6 pence of the given quantity, and obtain $9060 + 6 = 9066$, the whole number of pence in the whole of the given quantity, £37 „ 15*s*. „ 6*d*.

£.	*s*.	*d*.
37 „	15 „	6

$$
\begin{array}{rl}
37 & „\ 15\ „\ 6 \\
\times 20 & \\
\hline
740 & \\
+15 & \\
\hline
755 & \text{shillings} \\
\times 12 & \\
\hline
9060 & \\
+6 & \\
\hline
9066 & \text{pence} \\
\hline
\end{array}
$$

Ex. 2.—To reduce 9066 pence into pounds.

Here we first reduce the 9066 pence into shillings; and as a shilling is 12 times as valuable as a penny, we make the given number, 9066, 12 times as small, and obtain $9066 \div 12 = 775$, the equivalent number of shillings, with 6 pence remaining. We next reduce these 755 shillings into pounds; and as a pound is 20 times as valuable as a shilling, we make the number, 755, 20 times as small, and obtain $755 \div 20 = 37$, the equivalent number of pounds, with 15 shillings remaining, which, with the previous remainder, 6 pence, give us £37 „ 15*s*. „ 6*d*. = 9066 pence, the given quantity.

$$
\begin{array}{r}
12\,)\,9066 \\
\hline
20\,)\,755\ „\ 6 \text{ pence.} \\
\hline
£37\ „\ 15\ „\ 6 \\
\hline
\end{array}
$$

8. When a unit of the required name is not an exact number of times more or less valuable than a unit of the given name, we cannot reduce from the one name into the other directly; we therefore have recourse to an intermediate denomination, of which an exact number is contained in a unit of each of the others; and reducing the given number first into this intermediate denomination, afterwards reduce the number of this intermediate denomination into an equivalent number of the required name.

Thus to reduce guineas into pounds; as a pound is not an

exact number of times less than a guinea, we have recourse to the intermediate denomination, shillings, of which 20 are contained in a pound, and 21 in a guinea; and first reducing the guineas into shillings, afterwards reduce these shillings into pounds.

Ex.—To reduce 80 guineas into pounds.

Here, reducing the 80 guineas first into shillings, we have $80 \times 21 = 1680$, the equivalent number of shillings in 80 guineas; and afterwards reducing these shillings into the required name, pounds, we have $1680 \div 20 = 84$, the equivalent number of pounds in the given quantity, 80 guineas.

80 guineas
21
2,0)168,0 shillings
£84

Ex. 2.—To reduce £84 into guineas.

Here, first reducing the £84 into shillings, we have $84 \times 20 = 1680$ shillings, and reducing these shillings into the required name, guineas, we have $1680 \div 21 = 80$, the equivalent number of guineas in the given quantity, £84.

£.
84
20
21) 1680 (80 guineas
168
0

9. When a given quantity, consisting of several denominations, is to be reduced to an equivalent quantity also consisting of several denominations, we must reduce the given quantity, and also a unit of the required quantity, into one common name, and the quotient of the given quantity divided by this last number will be the equivalent quantity required.

Ex.—To reduce 54 coins, each worth 13*s*. „ 9½*d*. to an equivalent number of coins each worth 6*s*. „ 10¾*d*.

Here, reducing 13*s*. 9½*d*. into farthings and multiplying by 54, we get 35748, the number of farthings in the given quantity; and reducing 6*s*. 10¾*d*. into farthings, we get 331 farthings, the number of farthings contained in a unit of the required quantity; and now dividing 35748 by 331, we get the quotient 108, which is the equivalent number of the coins required.

```
                    s.   d.
                    13 „ 9¼
                    12
                    ---
                    165
                      4
    s.   d.         ---
     6 „ 10¾        662
    12               54
    --              ----
    82             2648
     4             3310
    ---            -----
    331   )        35748 ( 108 coins.
                   331
                   -----
                    2648
                    2648
                    ----
```

Note. In this example it may be remarked, that as the value of one of the required coins is just twice as small as the value of one of the given coins, the number must be just twice as great; consequently $54 \times 2 = 108$, the same answer as before; but as this is merely accidental, the general rule given above is requisite.

EXAMPLES FOR PRACTICE.

1. Reduce £357 into shillings. *Ans.* 7140 sh.
2. Reduce £357 „ 17*s.* into shillings. *Ans.* 7157 sh.
3. Reduce £357 „ 17*s.* „ 6*d.* into pence. *Ans.* 85890 d.
4. Reduce £357 „ 17*s.* „ 6¾*d.* into farthings. *Ans.* 343563 f.
5. Reduce 342720 farthings into pence, shillings, and pounds. *Ans.* 85680*d.* 7140*s.* £357.
6. Reduce 343536 farthings into pounds. *Ans.* £357 „ 17*s.*
7. Reduce 343560 farthings into guineas. *Ans.* 340 gui. 17*s.* „ 6*d.*
8. Reduce 345600 farthings into threepences, shillings, and pounds. *Ans.* 28800 threep. 7200 sh. £360.
9. Reduce 45360 farthings into groats, shillings, and guineas. *Ans.* 2835 grts. 945 sh. 45 gui.

10. Reduce 139968 farthings into sixpences, dollars, and moidores. *Ans.* 5832 sixp. 648 dol. 108 moid.

11. Reduce 4560 seven-shilling pieces into threepences, half-crowns, and pounds.

Ans. 127680 threp. 12768 hf. cr. £1596.

12. Reduce 324 quarter guineas into dollars and moidores. *Ans.* 378 dol. 63 moid.

13. Reduce 12345 English ells into quarters, nails, and inches. *Ans.* 61725 qrs. 246900 na. 555525 in.

14. Reduce 123456 yards into Flemish ells, French ells, and English ells.

Ans. 164608 Fl. ells, 82304 Fr. ells, 98764 Eng. ells, 4 qrs.

15. Reduce 175 miles into furlongs, poles, and yards.

Ans. 1400 fur. 56000 po. 308000 yds.

16. Reduce 75 miles 7 furlongs 5 poles into inches.

Ans. 4808430 in.

17. Reduce 14256000 barleycorns into miles and leagues.

Ans. 25 lea. 75 mi.

18. Reduce 11 lb. 9 oz. 15 dwts. 19 grs. troy into grains. *Ans.* 68059 grs.

19. Reduce 456789 grains into pounds troy.

Ans. 79 lb. 3 oz. 12 dwt. 21 grs.

20. Reduce 5 tons 19 cwts. 3 qrs. 27 lb. 15 oz. 14 drs. into drams avoirdupois. *Ans.* 3440638 dr.

21. Reduce 2345678 drams into tons.

Ans. 4 tons. 1 cwt. 0 qrs. 2 lb. 12 oz. 14 dr.

22. How many pounds troy are there in 234 lb. avoirdupois, allowing the pound avoirdupois to be equal to 14 oz. 11 dwts. 16 grs. troy?

Ans. 284 lbs. troy. 4 oz. 10 dwts.

23. In 75 pieces of cloth, each 25 ells Flemish, how many pieces, each containing 15 ells English?

Ans. 75 pieces

24. How many statute miles in 360 geographic miles, 69½ statute miles, and 60 geographic miles, making one degree? *Ans.* 417 stat. miles.

25. How many pounds avoirdupois in 350 lbs. troy?

Ans. 288 lbs. avoird.

26. Reduce 15 years into seconds.

Ans. 473040000 sec.

27. Reduce 1419120000 seconds into years.

Ans. 45 years.

28. How many minutes in 27 solar years, consisting of 365 days 5 hours 48 minutes 58 seconds?

Ans. 14200622 min. 6 sec.

29. Reduce 417 statute miles into geographic miles.

Ans. 360 geog. miles.

30. How many dozens of table-spoons, each spoon weighing 6 oz. 11 dwts. 8 grs. may be made from 10 ingots of silver, weighing each 16 lb. 5 oz?

Ans. 25 dozen.

31. How many dozens of salt-spoons, each spoon weighing 1 oz. 2 dwts.; tea-spoons, each 3 oz. 8 dwts.; and table-spoons each 4 oz. 16 dwts. may be made out of 55 lb. 9 oz. 12 dwts. of silver? *Ans.* 6 dozen.

32. Reduce 36 hhds. of wine into tierces, gallons, and pints. *Ans.* 54 tierces. 2268 gal. 18144 pts.

33. How many dozens of wine, each bottle containing one pint and a half, are there in 50 pipes?

Ans. 2800 dozen.

34. How many minutes have elapsed since the commencement of the present century to the present minute, allowing the year to be 365 days 6 hours?

Ans. Contingent depending on the time of commencing the calculation.

35. How many lunar months, consisting of 29 days 12 hours 44 minutes 3 seconds in solar years? one

Ans. 12 lu. mo. 10 days. 21 ho. 0 min. 22 sec.

36. How long would it require to count £150,000000, at the rate of £150 per minute without intermission?

Ans. 1 year, 329 days, 10 ho. 40 min.

37. How many square inches are contained in a square mile? *Ans.* 4014489600 sq. inches.

38. Reduce 4014489600 square inches into acres.

Ans. 640 acres, or 1 sq. mile.

39. How may cubic inches are contained in 127 cubic yards? *Ans.* 5925312 cub. inches.

40. In 29601560 cubic inches, how many cubic yards?

Ans. 63 cub. yds., 12 cub. feet., 96 cub. inches.

MAGNITUDE.

1. Magnitude may be considered either as positive or as relative.

2. Positive magnitude is that intrinsic magnitude which a quantity possesses in itself, independently of comparison with any other quantity.

3. Relative magnitude is that incidental magnitude which a quantity derives from comparison with another quantity of the same kind.

4. The positive magnitude of a quantity is expressed by the number representing it, as the positive magnitude of six units is expressed by the number 6.

5. The relative magnitude of a quantity is expressed by the quotient of the number representing it, divided by the number representing the quantity with which it is compared.

Thus the relative magnitude of the quantity 6, when compared with the quantity 3, is expressed by the quotient of 6 divided by 3, or $6 \div 3 = 2$.

6. The positive magnitude of the same quantity is always the same; thus, the positive magnitude of the quantity six units is always 6.

7. The relative magnitude of the same quantity, varies according to the magnitude of the quantity with which it is compared; becoming great when compared with a quantity which is less, and less when compared with a quantity which is greater than itself.

Thus the relative magnitude of 6, when compared with 3, is $6 \div 3 = 2$; and the relative magnitude of 6, when compared with 2, is $6 \div 2 = 3$; here it is evident that the relative magnitude of 6 is great when compared with 3, and greater still when compared with 2.

Also the relative magnitude of 6, when compared with 12, is $6 \div 12 = \frac{1}{2}$; and the relative magnitude of 6 when

compared with 18, is $6 \div 18 = \frac{1}{3}$; here it is evident that the relative magnitude of 6 is small when compared with 12, and smaller still when compared with 18.

EXERCISES IN RELATIVE MAGNITUDE.

1. What is the relative magnitude of 27 when compared with 9?

2. What is the relative magnitude of 27 when compared with 3?

3. Is the relative magnitude of 27, when compared with 9, greater or less than when compared with 3?

4. What is the relative magnitude of 9 when compared with 27?

5. What is the relative magnitude of 3 when compared with 27?

6. Which is greater, the relative magnitude of 9 when compared with 27, or of 3 when compared with 27?

7. Is the relative magnitude of 105 compared with 15 greater or less than the relative magnitude of 63 compared with 9?

8. Which is greater, the relative magnitude of 126 compared with 21, or the relative magnitude of 27 when compared with 9?

9. Which is greater, the relative magnitude of 11 when compared with 55, or the relative magnitude of 15 compared with 150?

10. How many times is the relative magnitude of 132 compared with 11 greater than the relative magnitude of 108 compared with 27?

VARIATION.

1. Variation is that change in the relative magnitude of a quantity, which necessarily results from its connexion with or dependence upon another quantity.

2. When two quantities are so mutually connected with or dependent upon each other, that a change cannot be made in the magnitude of the one without producing a change in the magnitude of the other also, these quantities are said to vary.

Thus, if the quantity of goods purchased depend upon the sum of money laid out, it is evident that if the sum of money be made either greater or less, the quantity of goods must also be made greater or less.

3. Quantities may vary either *directly* or *inversely*, accordingly as the change made in the magnitude of the one, may require either a similar or a contrary change to be made in the magnitude of the other.

4. Two quantities are said to vary *directly*, if when one of them be made greater, the other must also be made greater; or if when one of them be made less, the other must also be made less.

Thus, when the quantity of goods purchased depends upon the sum of money laid out, it is evident that if the sum of money be made greater, the quantity of goods must also be made greater; or if the sum of money be made less, the quantity of goods must also be made less.

Hence, the quantity of goods varies directly as the sum of money; for the change made in the magnitude of the one, necessarily produces a similar change in the magnitude of the other; and for this reason the sum of money will also vary directly as the quantity of goods.

5. Two quantities are said to vary *inversely*, if when one of them be made greater, the other must consequently be made less; or if when one of them be made less, the other must consequently be made greater.

Thus, when the quantity of goods purchased depends upon the price, it is evident that if the price be made greater, the quantity of goods must consequently be made less; or if the price be made less, the quantity of goods must consequently be made greater.

Hence the quantity of goods varies inversely as the price; for the change made in the magnitude of the one, necessarily produces a contrary change in the magnitude of the other; and for this same reason the price will also vary inversely as the quantity of goods.

A very little exercise of the judgment will ascertain in what manner dependent quantities will vary; a child scarcely requires to be told that, in buying oranges at one penny each, he will get a greater number for sixpence than for threepence; or that for sixpence he will have twice as

many oranges at one penny each, as he will have at twopence each.

EXERCISES IN VARIATION.

1. In purchasing goods, if the price remain always the same, will the quantity of goods vary directly or inversely as the sum of money?

2. If the quantity of goods remain always the same, will the sum of money vary directly or inversely as the price?

3. If the sum of money remain always the same, will the quantity of goods vary directly or inversely as the price?

4. In performing any work, if the time remain always the same, will the quantity of work vary directly or inversely as the number of men employed?

5. If the number of men remain unaltered, will the quantity of work vary directly or inversely as the time?

6. If the quantity of work remain unaltered, will the time vary directly or inversely as the number of men?

7. In travelling any distance, if the time be the same, will the distance vary directly or inversely as the speed or rate of travelling?

8. If the speed remain the same, will the time vary directly or inversely as the distance?

9. If the distance remain the same, will the time vary directly or inversely as the speed?

10. In the Rule of Multiplication, if the multiplicand remain the same, will the product vary directly or inversely as the multiplier?

11. If the product remain the same, will the multiplier vary directly or inversely as the multiplicand?

12. In the Rule of Division, if the dividend remain the same, will the quotient vary directly or inversely as the divisor?

RATIO.

1. Ratio is the relative magnitude which one quantity has in comparison with another of the same kind; and is ascertained by finding how many times the one quantity is greater or less than the other.

Thus we may compare one weight with another weight, or one sum of money with another sum of money, and ascertain its relative magnitude by finding how many times it is greater or less than the other.

But we can form no idea of the relative magnitude of a hundred weight and a mile, or of a bushel and an hour; for we cannot compare things so totally dissimilar, neither can we find how many times the one is greater or less than the other.

Hence it is obvious that ratio can subsist only between two quantities which are both of the same kind.

2. The ratio of two quantities is expressed by placing two points (:) between them; thus the ratio of 6 to 3 is written 6 : 3; the former of these quantities is called the antecedent of the ratio, and the latter is called the consequent.

3. Of the two quantities compared, that is always the antecedent concerning which something is known, and that is the consequent concerning which something is required; and it is only by thus distinguishing the antecedent from the consequent, that the ratio between the two quantities is determined.

Thus, if the value of 6 gallons be known, and the value of 3 gallons be required, 6 will be the antecedent, and the ratio will be 6 : 3; but if the value of 3 gallons be known, and the value of 6 gallons be required, 3 will be the antecedent, and the ratio will be 3 : 6.

4. The antecedent and the consequent must be perfectly alike in every respect, except in that of magnitude; otherwise the ratio of the numbers, will not express the true ratio of the quantities they represent.

Thus, the relative magnitude of one gallon to one pint will not be truly expressed by the ratio 1 : 1; for one gallon is equal to eight pints, consequently the relative

magnitude of these quantities can only be properly expressed by the ratio 8 : 1.

Hence, when the antecedent and consequent, though both of the same kind, are of different denominations, they must both be reduced into one and the same denomination.

5. When the antecedent is equal to its consequent, the ratio is called a ratio of equality; as the ratio 6 : 6, 3 : 3, &c.

When the antecedent is greater than, or a multiple of, its consequent, the ratio is called a ratio of greater inequality; as the ratios 6 : 3, 8 : 4, &c.; and

When the antecedent is less than, or a part of its consequent, the ratio is called a ratio of less inequality; as the ratios 3 : 6, 4 : 8, &c.

6. The antecedent is said to be a multiple of the consequent when it contains the consequent any certain number of times exactly without a remainder; as in the ratio 6 : 3, 6 is a multiple of 3, containing it twice.

The antecedent is said to be a part of the consequent when it is contained in the consequent any certain number of times exactly without a remainder; as in the ratio 3 : 6, 3 is a part of 6, being contained in it twice.

7. The measure of a ratio is expressed by the quotient of the antecedent, divided by the consequent; thus, the measure of the ratio 6 : 3 is $6 \div 3 = 2$, and the measure of the ratio 3 : 6 is $3 \div 6 = \frac{1}{2}$.

8. Two ratios are equal when the antecedent of the one is the same multiple, or the same part of its consequent, as the antecedent of the other ratio is of its consequent.

> Thus the ratio 6 : 3 is equal to the ratio 8 : 4; for $6 \div 3 = 2$, and $8 \div 4 = 2$; also the ratio 3 : 6 is equal to the ratio 4 : 8, for $3 \div 6 = \frac{1}{2}$, and $4 \div 8 = \frac{1}{2}$.

Of two ratios, that is the greater, of which the antecedent is the greater multiple, or the greater part of its consequent.

> Thus, the ratio 6 : 2 is greater than the ratio 8 : 4, for $6 \div 2 = 3$, and $8 \div 4 = 2$; also the ratio 4 : 8 is

greater than the ratio $2:6$, for $4 \div 8 = \frac{1}{2}$, and $2 \div 6 = \frac{1}{3}$.

9. If the antecedent and consequent of a ratio be both multiplied by the same number, the ratio is not altered.

Thus, if the antecedent and consequent of the ratio $6:3$ be both multiplied by 3, then $\overline{6 \times 3} : \overline{3 \times 3}$, or $18:9$ is the same as the ratio $6:3$; for $6 \div 3 = 2$; and $18 \div 9 = 2$.

Also, if the antecedent and consequent of the ratio $3:6$ be both multiplied by 3, then $\overline{3 \times 3} : \overline{6 \times 3}$, or $9:18$ is the same as the ratio $3:6$; for $3 \div 6 = \frac{1}{2}$, and $9 \div 18 = \frac{1}{2}$.

10. If the antecedent and consequent of a ratio be both divided by the same number, the ratio is not altered.

Thus, if the antecedent and consequent of the ratio $6:3$ be both divided by 3, then $\overline{6 \div 3} : \overline{3 \div 3}$, or $2:1$, is the same as the ratio $6:3$, for $6 \div 3 = 2$, and $2 \div 1 = 2$.

Also, if the antecedent and consequent of the ratio $3:6$ be both divided by 3, then $\overline{3 \div 3} : \overline{6 \div 3}$, or $1:2$, is the same as the ratio $3:6$, for $3 \div 6 = \frac{1}{2}$, and $1 \div 2 = \frac{1}{2}$.

Hence any ratio may be expressed in lower terms by dividing the antecedent and consequent by any number that will divide them both without a remainder.

11. If the antecedents of several ratios be all multiplied together, and also their consequents, the ratio between the product of the antecedents and the product of the consequents, will be a ratio compounded of all the ratios between the several antecedents and their respective consequents.

Thus, if we multiply together the antecedents of the ratios $6:3$ and $12:4$, and also their consequents, we shall have $6 \times 12 = 72$, the product of the antecedents, and $3 \times 4 = 12$, the product of the consequents; and the ratio $72:12$ will be the ratio compounded of the two ratios $6:3$ and $12:4$.

Also, if we multiply together the antecedents of the ratios $3:6$ and $4:12$, and also their consequents, we shall have $3 \times 4 = 12$, the product of the ante-

cedents, and $6 \times 12 = 72$, the product of the consequents; and the ratio 12 : 72 will be the ratio compounded of the two ratios 3 : 6 and 4 : 12.

In this compound ratio, the antecedent will always be either such a multiple or such a part of the consequent, as is equal to the product of all the multiples or of all the parts, which each of the antecedents is of its consequent in the several ratios compounded.

Thus, in the ratio 6 : 3, the antecedent 6 is twice as great as its consequent 3, and in the ratio 12 : 4, the antecedent 12 is three times as great as its consequent 4; and the product of these multiples, $2 \times 3 = 6$, is equal to the multiple, which in the compound ratio 72 : 12, the antecedent 72 is of the consequent 12, for $72 \div 12 = 6$.

Also, in the ratio 3 : 6, the antecedent 3 is one half of its consequent 6, and in the ratio 4 : 12 the antecedent 4 is one-third part of its consequent 12; and the product of these parts $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, is equal to the part which in the compound ratio 12 : 72, the antecedent 12 is of the consequent 72, for $12 \div 72 = \frac{1}{6}$.

If we had reduced each of these ratios into its lowest terms before we compounded them (Art. 10), we should have obtained the compound ratio in its lowest terms.

Thus, dividing the antecedent and consequent of the ratio 6 : 3 by 3, we should have had the equal ratio 2 : 1; and dividing the antecedent and consequent of the ratio 12 : 4 by 4, we should have had the equal ratio 3 : 1, and compounding these quotients, have obtained the compound ratio 6 : 1, which is the same as the ratio 72 : 12, for $72 \div 12 = 6$, and $6 \div 1 = 6$.

$$\begin{array}{c}
2 : 1 \\
\cancel{6} : \cancel{3} \\
3 : 1 \\
\cancel{12} : \cancel{4} \\
\hline
6 : 1
\end{array}$$

Also, in compounding the ratios 3 : 6 and 4 : 12, dividing as before, we should have had the ratios 1 : 2 and 1 : 3 respectively equal to the former; and compounding these, have obtained the compound ratio 1 : 6, which is the same as the ratio 12 : 72, for $12 \div 72 = \frac{1}{6}$, and $1 \div 6 = \frac{1}{6}$.

$$\begin{array}{c}
1 : 2 \\
\cancel{3} : \cancel{6} \\
1 : 3 \\
\cancel{4} : \cancel{12} \\
\hline
1 : 6
\end{array}$$

In reducing the several ratios to be compounded, we may divide any one of the antecedent and any one of the consequents whatever; for as their several products form the antecedent and consequent of the compound ratio, if both collectively are divided by the same numbers, the ratio will be the same.

Thus, in compounding the ratios 5 : 7 and 14 : 15, as we cannot divide 5 and 7 by the same number, we may divide the antecedent 5 of that ratio and the consequent 15 of the other ratio, both by 5; and the antecedent 14 and the consequent 7 both by 7, and compounding these quotients, we shall obtain the compound ratio 2 : 3, which is the same as the ratio 70 : 105, or the compound ratio of the undivided numbers, for $70 \div 105 = \frac{2}{3}$, and $2 \div 3 = \frac{2}{3}$.

$$\begin{array}{ccc} 1 & : & 1 \\ \cancel{5} & : & \cancel{7} \\ 2 & : & 3 \\ \cancel{14} & : & \cancel{15} \\ \hline 2 & : & 3 \end{array}$$

12. A ratio of greater inequality compounded with another is made greater; and a ratio of less inequality compounded with another is made less.

Thus if the ratio 6 : 3 be compounded with the ratio 8 : 4 then $\overline{6\times8} : \overline{3\times4}$ or 48 : 12 is greater than the ratio 6 : 3, for $48 : 12 = 48 \div 11 = 4$; and $6 : 3 = 6 \div 3 = 2$.

Also if the ratio 3 : 6 be compounded with the ratio 4 : 8, then $\overline{3\times4} : \overline{6\times8}$ or 12 : 48 is less than the ratio 3 : 6; for $12 : 48 = 12 \div 48 = \frac{1}{4}$; and $3 : 6 = 3 \div 6 = \frac{1}{2}$.

13. A ratio of greater inequality compounded with a ratio of less inequality is made less; and a ratio of less inequality compounded with a ratio of greater inequality is made greater.

Thus, if the ratio 6 : 3 be compounded with the ratio 4 : 8, then $\overline{6\times4} : \overline{3\times8}$, or 24 : 24 is less than the ratio 6 : 3; for $24 : 24 = 24 \div 24 = 1$, and $6 : 3 = 6 \div 3 = 2$.

Also if the ratio 3 : 6 be compounded with the ratio 8 : 4, then $\overline{3\times8} : \overline{6\times4}$, or 24 : 24 is greater than the ratio 3 : 6; for $24 : 24 = 24 \div 24 = 1$, and $3 : 6 = 3 \div 6 = \frac{1}{2}$.

14. If several ratios be compounded in which the con sequent of the first ratio is equal to the antecedent of the second; the consequent of the second to the antecedent of

the third, and so on throughout, the compound ratio will be the ratio of the first antecedent to the last consequent.

Thus, if we compound the ratios 3 : 5, 5 : 7 and 7 : 21, we shall have the compound ratio 3 : 21, which is the ratio of the first antecedent 3, to the last consequent 21.

$$\begin{array}{r}3 : \cancel{5} \\ \cancel{5} : \cancel{7} \\ \cancel{7} : 21 \\ \hline 3 : 21\end{array}$$

15. A ratio is said to be inverted, when the antecedent and consequent are mutually interchanged; thus the ratio 6 : 3, when inverted, becomes the ratio 3 : 6; and 4 : 8 is the inverted ratio of 8 : 4.

EXERCISES IN RATIO.

1. What is the measure of the ratio 45 : 9, and also of the ratio 9 : 45?

2. What is the measure of the ratio 81 : 3, and how many times is it greater than the ratio 45 : 5?

3. What is the measure of the ratio 3 : 81, and how many times is it greater or less than the ratio 5 : 45?

4. How many times is the ratio 132 : 11 greater than the ratio 54 to 9?

5. How many times is the ratio 35 : 105, greater or less than the ratio 11 : 165?

6. Compound the ratios 3 : 27 and 5 : 45.

7. Reduce into their lowest terms, and compound the ratios, 132 : 11, and 55 : 660.

8. Reduce into their lowest terms, and compound the ratios, 21 : 11, 11 : 35, 35 : 7, 7 : 28, and 28 : 16.

9. Compound the ratios 3 : 5, 5 : 17, 17 : 9, 9 : 25, and 25 : 8.

10. Which is greater, the ratio of 8 : 7 or the ratio 9 : 8?

11. How many times is the ratio compounded of the ratios 45 : 9 and 24 : 8, greater or less than the ratio compounded of the ratios 81 : 27 and 125 : 5?

12. How many times is the inverted ratio 11 : 165 compounded with the ratio 27 : 81, greater or less than the ratio 11 : 165 compounded with the inverted ratio 81 : 27.

PROPORTION.

When of four quantities taken in any order, the third is as many times greater or as many times less than the fourth, as the first is greater or less than the second, the third is said to bear to the fourth, the same proportion as the first does to the second.

1. Proportion, then, is that relation in respect of magnitude, which subsists between four quantities, when the ratio of the first to the second is the same as the ratio of the third to the fourth.

Thus, the quantities 6, 3, 8, and 4, taken in that order, are proportionals; for the ratio $6 : 3 = 2$ is the same as the ratio $8 : 4 = 2$; also the quantities 3, 6, 4 and 8 are proportionals; for the ratio $3 : 6 = \frac{1}{2}$, is the same as the ratio $4 : 8 = \frac{1}{2}$.

2. The proportion of four quantities is usually expressed by placing two points (:) between the first and second, and also between the third and fourth of the quantities; and four points (: :) between the second and third.

Thus the proportion of the four quantities 6, 3, 8 and 4, is written $6 : 3 :: 8 : 4$, denoting that 6 bears to 3 the same proportion as 8 does to 4, or that 6 is to 3 as 8 is to 4; the first and fourth of these quantities are called extremes; and the second and third are called means.

3. If four quantities are proportionals, the product of the extremes will be equal to the product of the means; thus if $6 : 3 :: 8 : 4$, $6 \times 4 = 3 \times 8$; for $6 \times 4 = 24$, and $3 \times 8 = 24$; also if $3 : 6 :: 4 : 8$, $3 \times 8 = 6 \times 4$; for $3 \times 8 = 24$ and $6 \times 4 = 24$.

For it is evident from the definition of proportion that if one of the extremes be greater than one of the means, the other extreme must be just so many times less than the other mean; or if the one extreme be less than the one mean, the other extreme must be just so many times greater than the other mean, and consequently their products must be the same.

Thus if $6 : 3 :: 8 : 4$, then $6 \times 4 = 3 \times 8$; for the first extreme 6, which is twice as great as the first mean 3, is multiplied by the other extreme 4, which is twice as small as the other mean 8; also if $3 : 6 :: 4 : 8$,

then $3\times8=6\times4$; for the first extreme 3, which is twice as small as the first mean 6, is multiplied by the other extreme 8, which is twice as great as the other mean 4.

4. If four quantities are proportionals, the sum of the first and second is to the second, as the sum of the third and fourth is to the fourth; thus, if $6:3::8:4$, then $\overline{6+3}:3::\overline{8+4}:4$, or $9:3::12:4$; for $9\div3=3$, and $12\div4=3$.

5. If four quantities are proportionals, the difference of the first and second is to the second, as the difference of the third and fourth is to the fourth; thus if $6:3::8:4$, then $\overline{6-3}:3::\overline{8-4}:4$; or $3:3::4:4$, which requires no demonstration.

6. If four quantities are proportionals, the first is to the sum of the first and second as the third is to the sum of the third and fourth; thus if $6:3::8:4$, then $6:\overline{6+3}::8:\overline{8+4}$; or $6:9::8:12$; for $6\div9=\frac{2}{3}$, and $8\div12=\frac{2}{3}$.

7. If four quantities are proportionals, the first is to the difference of the first and second, as the third is to the difference of the third and fourth; thus if $6:3::8:4$, then $6:\overline{6-3}::8:\overline{8-4}$, or $6:3::8:4$; for $6\div3=2$, and $8\div4=2$.

8. If four quantities are proportionals, the sum of the first and second is to their difference, as the sum of the third and fourth is to their difference; thus, if $6:3::8:4$, then $\overline{6+3}:\overline{6-3}::\overline{8+4}:\overline{8-4}$, or $9:3::12:4$, for $9\div3=3$, and $12\div4=3$.

9. From the equality of the products of the extremes and means of proportional quantities, is usually derived the rule for finding the fourth term of a proportion, when three terms only are given; and which is thence called the "*Rule of Three.*"

For of the three given terms, two are always means, and the other is an extreme; and as the product of the extremes is always equal to the product of the means, it is evident that if this product be divided by the given extreme, the quotient will be the other extreme or fourth term required.

10. But independently of this property, we may derive this same rule from the definition of Proportion alone; for as the ratio of the third term to the fourth, is the same as the ratio of the first to the second; we have only to consider the third term as an antecedent, of which the consequent is to be found in that ratio.

For this purpose we have only to multiply the third term by the consequent of the ratio, and to divide the product by the antecedent; and the quotient will be the consequent required, or the fourth term of the proportion.

Thus, to find the fourth term of the proportion of which the first three terms 6, 3, 8 only are given, we have only to find a consequent to the antecedent 8 in the ratio $6 : 3$; and $\overline{8 \times 3} \div 6$, or $24 \div 6 = 4$, is the consequent required, or the fourth term of the proportion; for $6 : 3 :: 8 : 4$.

Also, to find the fourth term of the proportion of which the first three terms, 3, 6, and 4, only are given, we have only to find a consequent to the antecedent 4 in the ratio $3 : 6$; and $\overline{4 \times 6} \div 3$, or $24 \div 3 = 8$, is the consequent required, or the fourth term of the proportion; for $3 : 6 :: 4 : 8$.

Here it is obvious that if the consequent of the given ratio be greater than the antecedent, the multiplier will be just so many times greater than the divisor, and the fourth term consequently so many times greater than the third.

Or if the consequent of the given ratio be less than the antecedent, the multiplier will be just so many times less than the divisor, and the fourth term consequently so many times less than the third.

Hence, in every case, the fourth term thus found, will be just as many times greater or as many times less than the third, as the second term is greater or less than the first; and consequently will be in the same proportion.

11. These principles are sufficient for the solution of all questions in Arithmetic; for no question can be proposed which requires anything more than either the increase or the decrease of a given quantity, according to the increase or decrease of other quantities in the question, on which its required magnitude may be dependent.

To find the required magnitude of this given quantity,

is to find the answer to the question; and for this purpose, the law of Variation ascertains whether the given quantity is to be increased or decreased; and that of Ratio determines precisely to what extent the required increase or decrease is to be made.

Thus, if the quantity £5 be given as the value of 7 gallons of wine, and the question require the value of 42 gallons—

First, it will be evident from the law of Variation that the sums of money must vary directly as the quantities of wine; and as the quantity of which the value is required is greater than the quantity of which the value is given, it is evident that the given sum of money, £5, must be increased.

Secondly, as the relative magnitude of the quantities of wine is expressed by the ratio $7 : 42$, and as the relative magnitude of the sums of money must be the same, it is evident that the given quantity, £5, must be increased in the ratio $7 : 42 = 1 : 6$, or be increased 6 times.

In applying these principles to practical use, we regard all proportion as founded upon ratio; for proportion, in fact, is only the equality of two ratios, requiring that the ratio of the third term to the fourth, shall be equal to the ratio of the first term to the second.

We shall conclude, that what is called the "*Single Rule of Three Direct*" is this same proportion, founded on the one ratio only which is given in the question, taken directly.

That what is called the "*Rule of Three Inverse*," or "*Inverse Proportion*," is the same proportion, founded on the ratio given in the question, taken inversely, or inverted.

That what is called the "*Rule of Five*," or "*Double Rule of Three*," is the same proportion, founded on the ratio compounded of all the several ratios given in the question, taken either directly or inversely, accordingly as the required magnitude of the given quantity may vary directly or inversely as the quantities of each ratio separately considered.

And lastly, that the same general rule for finding the fourth term of the proportion, by multiplying the second and third terms together, and dividing their product by

the first term, is, without any deviation, equally applicable to all.

12. Every question will contain, besides the given quantity to be increased or decreased, as many ratios as it has separate conditions; thus, if it have only one condition, it will contain 3 terms; if two conditions, 5 terms; if three conditions, 7 terms, &c., &c.

But whatever may be the number of terms contained in the question, every two terms which are of the same kind will express some particular condition, subject to which the given quantity is to be increased or decreased; and the ratio between them will show the exact amount of the increase or decrease required by that particular condition.

The given quantity which is to be increased or decreased may be easily distinguished from all the other terms by its having no corresponding number of the same kind in the question; and will always be the third term of the proportion, or the antecedent to the fourth term or answer required.

The various rules under which questions in Arithmetic are usually classed, are only the mere names of the various purposes to which these simple principles are indiscriminately applicable; and from which, for the solution of all questions, may be deduced the following

GENERAL RULE.

1. Reserve for the third term of the proportion, that quantity which has no corresponding number of the same kind in the question; and if there be only three terms, find the given ratio of the other two, which are always of the same kind.

2. If the quantities of which one is required, vary directly, as those of the given ratio, write the ratio as it is given; but if they vary inversely, invert the given ratio; and the antecedent and consequent will be the first and second terms of the proportion.

3. If there be several ratios in the question, write them all down under each other, either directly or inversely, accordingly as the quantities of which

one is required, may vary directly or inversely, as the quantities of each ratio separately considered.

4. Reduce into their lowest terms, and compound the several ratios; and the antecedent and consequent of the compound ratio, will be the first and second terms of the proportion, to which annex with four points (: :) the quantity reserved for the third term.

5. Lastly, multiply together the second and third terms, and divide their product by the first term; and the quotient will be the fourth term of the proportion, or answer to the question; and will be always of the same denomination as the third term, of which it is always either a multiple or a part.

Ex. 1.—If 24 ounces of silver cost £6, what must be paid for 56 ounces at the same rate?

$$\begin{array}{lcccccccc} \text{Given ratio} & & & & & & \text{£.} & & \text{£.} \\ \text{of silver} & \cancel{24} & : & \cancel{56} & :: & & 6 & : & 14 \\ & 3 & & 7 & & & 7 & & \\ & & & & & 3) & 42 & & \\ & & & & & & 14 & & \end{array}$$

Here, as £6 has no corresponding number of the same kind in the question, it is reserved for the third term of the proportion; and as of the two other terms in the question, we know the value of 24 ounces, 24 is the antecedent, and the given ratio is 24 : 56. Again, as we know that, according to the law of Variation, the sums of money of which one is required, must vary directly as the quantities of silver, we write down the given ratio of silver, 24 : 56, of which the antecedent and consequent are the first and second terms of the proportion; and now annexing with four points, the quantity £6, which was reserved for the third term, we have the three terms of the proportion properly stated. We have here reduced the ratio 24 : 56 into its lowest terms 3 : 7, by dividing both the antecedent and consequent of the given ratio by 8; and now multiplying the second and third terms, 7 and 6, and dividing their product, 42, by the first term, 3, we have the quotient, 14, which is the fourth term of the proportion, or the answer to the question.

Ex. 2.—If 24 ounces of silver, at 5*s*. per ounce, may be bought for £6, how many ounces of silver, at 7*s*. 6*d*. per ounce, can I have for the same money?

In this question every process is the same as in the last example, till we have ascertained the given ratio to be 5 : 7*s.* 6*d.*; but as we know from the law of Variation, that as the sum of money remains the same, the quantities of silver must vary inversely as the prices, we write down the inverted ratio of prices, 7*s.* 6*d.* : 5, of which the antecedent and consequent are the first and second terms of the proportion; but as the first term contains 7 shillings and sixpence, and the second 5 shillings only, we reduce both into sixpences that they may be both of the same name; and now annexing the third term 24 ounces, as before, we multiply the second and third terms together, and dividing their product, 240, by the first term, 15, we get the quotient, 16 ounces, which is the fourth term of the proportion, or the answer to the question.

```
Inverted ratio                     oz.  oz.
  of prices    7s. 6d. : 5 :: 24 : 16
                  2      2
                 ---    ---
                 15     10
                        24
                       ---
                 15 ) 240 ( 16
                      15
                      ---
                       90
                       90
                       ---
```

Ex. 3.—If £120 will supply a family of 6 persons for 9 months, for how many months will £240 supply a family of 18 persons?

In this question 9 months is the only quantity which has no other of the same kind given; we therefore reserve it for the third term of the proportion; and of the other quantities in the question, we select the two, £120 and £240, which are of the same kind, and find their given ratio to be £120 : 240; and as the quantities, months, of which one is required, will vary directly as the sums of money which are the quantities of this ratio, we write down the given ratio of money 120 : 240. We next select

$$\begin{array}{lccccccc}
\text{Given ratio of} & 1 & : & 2 & & & & \\
\quad \text{money} \;.\;. & \cancel{120} & : & \cancel{240} & & & & \\
\text{Inverted ratio} & 3 & : & 3 & & & & \\
\quad \text{of persons}\;. & \cancel{18} & : & \cancel{6} & & & & \\
\hline
 & & & & & \text{mo.} & & \text{mo.} \\
\text{Compound ratio} & 3 & : & 2 & :: & 9 & : & 6 \\
 & & & & & 2 & & \\
 & & & & 3\,) & \overline{18} & & \\
 & & & & & \overline{6} & &
\end{array}$$

the two quantities, 6 persons and 18 persons, which are of the same kind, and find their given ratio to be 6 : 18; but as we know from the law of Variation that the months must vary inversely as the persons, or the quantities of this ratio, we write down the inverted ratio of persons, 18 : 6, underneath the ratio of money, and drawing a line under them, we reduce these ratios into their lowest terms, 1 : 2 and 3 : 1, and compounding these, obtain the compound ratio 3 : 2, of which the antecedent and consequent are the first and second terms of the proportion; and now annexing the third term, 9 months, which was reserved for that purpose, and multiplying the second and third terms together, and dividing their product, 18, by the first term, 3, we get the quotient, 6 months; which is the fourth term of the proportion, or the answer to the question.

EXAMPLES FOR PRACTICE.

1. If 3 lb. of tea cost 18*s*., what will 36 lb. cost at the same price? *Ans*. £10 16*s*.

2. If 36 lb. of tea cost £10 16*s*., how many lbs. can be bought for 18*s*.? *Ans*. 3 lb.

3. If 36 lb. of tea cost £10 16*s*., what must be paid for 3 lb.? *Ans*. 18*s*.

4. What is the value of 1 cwt. of sugar, at 10½*d*. per lb.? *Ans*. £4 18*s*.

5. What is the value of 1 chaldron of coals, at 1*s*. 6½*d*. per bushel? *Ans*. £2 15*s*. 6*d*.

6. If a firkin of butter weighing 56 lb. cost £2 9*s*., what must be paid for 1 lb.? *Ans*. 10½*d*.

7. If 84 lb. of cheese cost 3 guineas, what will be the value of 21 lb.? *Ans*. 15*s*. 9*d*.

8. If 10½ lb. of cheese cost 8*s*. 9*d*., what will be the value of 2 cwt. 2 qrs.? *Ans*. £11 13*s*. 4*d*.

9. If 375 men require 3250 rations of bread per month, how many rations will serve 3375 men? *Ans*. 29250.

10. If 3250 rations will serve 375 men for 1 month, how long will the same quantity serve 75 men? *Ans*. 5 mo.

11. If 365 men dig 2555 yards of earth in 1 day, how many yards will 73 men dig in the same time?

Ans. 511 yds.

12. What is the value of $2\frac{1}{2}$ cwt. of coffee, at $5\frac{1}{2}d.$ per oz.? *Ans.* £102 13*s.* 4*d.*

13. What is the value of 375 yards of cloth, at 3*s.* 6*d.* per ell Flemish? £87 10*s.*

14. If 2 cwt. 3 qr. 14 lb. of chocolate cost £53 13*s.* 4*d.*, what must be paid for 3 qr. 14 lb.? *Ans.* £16 6*s.* 8*d.*

15. If 11 yards of cloth cost £2 15*s.*, what must be paid for 2 qr. 2 nails? *Ans.* 3*s.* $1\frac{1}{2}d.$

16. What must be paid for 3 pieces of muslin, each containing 35 ells Flemish, at 5*s.* 3*d.* per ell English?

Ans. £16 10*s.* 9*d.*

17. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at £16 4*s.* per piece; required the value of the whole, and the price per yard?

Ans. £388 16*s.* val. of whole; 12*s.* per yard.

18. Bought 5 bales of cloth, each 15 pieces, and each piece 25 English ells, for £546 17*s.* 6*d.*; what is the value of one ell Flemish? *Ans.* 3*s.* 6*d.*

19. If 25 men will do a piece of work in 12 days, how many will do the same in 4 days? *Ans.* 75 men.

20. If 30 men will do a piece of work in 18 days, in what time will 45 men do the same? *Ans.* 12 days.

21. If a journey be performed in 12 days, at the rate of 6 miles per hour, in what time might the same be done at the rate of 9 miles per hour? *Ans.* 8 days.

22. If a journey be performed in 18 days, at the rate of 8 miles per hour, at what rate per hour could it be done in 12 days? *Ans.* 12 miles per hour.

23. If a yard of stuff cost 3*s.* 4*d.*, what will be the value of $52\frac{1}{2}$ English ells? *Ans.* £10 19*s.* $9\frac{1}{2}d.$

24. What will be the tax upon a rental of £745 15*s.* 6*d.*, at 3*s.* 6*d.* in the £.? *Ans.* £130 10*s.* $2\frac{1}{2}d.$, $\frac{1}{4}$.

25. If upon £1260, an assessment of £105 be made, at what rate is that in the £.? *Ans.* 1*s.* 8*d.*

26. How much tea, at 8*s.* per lb., can I have for 2 cwt. of coffee at 2*s.* 6*d.* per lb.? *Ans.* 70 lb.

27. How much coffee, at 2*s.* 6*d.* per lb., can I have for 70 lb. of tea at 8*s.* per lb.? *Ans.* 2 cwt.

28. If for 70 lb. of tea, at 8*s.* per lb., I receive 2 cwt. of coffee, what is the price of the coffee per lb.?
Ans. 2*s.* 6*d.* per lb.

29. If for 2 cwt. of coffee, at 2*s.* 6*d.* per lb., I receive 70 lb. of tea, what was the price of the tea per lb.?
Ans. 8*s.* per lb.

30. What is the value of 3 casks of raisins, each weighing as follows:—No. 1, 2 cwt. 3 qrs. 14 lb.; No. 2, 3 cwt. 0 qrs. 15 lb.; and No. 3, 1 cwt. 3 qrs. 27 lb., at $5\frac{3}{4}$*d.* per lb.?
Ans. £21 9*s.* 4*d.*

31. What is the value of 15 hhds. of rum, at £28 per tierce? *Ans.* £630.

32. How much in length, that is $4\frac{1}{2}$ inches broad, will make a square foot, which is 12 in. long and 12 in. broad?
Ans. 32 inches.

33. How much in breadth, that is 16 in. long, will make a square foot? *Ans.* 9 inches.

34. How much in length, that is $2\frac{3}{4}$ yards wide, will make a square pole? *Ans.* 11 yds.

35. How much in breadth, that is 11 yards in length, will make a square pole? *Ans.* $2\frac{3}{4}$ yds.

36. How many yards of matting, 2 feet 6 in. wide, will cover a floor that is 54 feet long and 40 feet wide?
Ans. 288 yds.

37. If 3 yards $\frac{3}{4}$ qrs. of cloth $1\frac{1}{2}$ yard wide will make a dress, how many yards will it require of cloth 1 yard 1 qr. wide? *Ans.* $4\frac{1}{2}$ yds.

38. How many yards of shalloon, 3 qrs. of a yard wide, will line a cloak which is 5 yards in length and $2\frac{1}{4}$ yards in breadth? *Ans.* 15 yds.

39. If 32 bricks will pave an area 1 yard square, how many will pave an area 35 feet long and 27 feet wide? *Ans.* 3360 brks.

40. If a principal of £100 gain £5 interest in 1 year, at the rate of 5 per cent., what principal will gain £37 10*s.* in the same time? *Ans.* £750.

41. If £750 gain £37 10*s.* interest in 1 year at 5 per cent., in what time will it gain £18 15*s.*? *Ans.* 6 mths.

42. At what rate per cent. did £250 gain £15 interest in 1 year? *Ans.* 6 per cent.

43. If a family of 15 persons spend £360 in 8 months, how much will supply a family of 24 persons for 5 months? *Ans.* £360.

44. If £180 will supply a family of 24 persons for 4 months, how long will £540 supply a family of 16 persons? *Ans.* 18 mths.

45. If 30 men build 24 houses in 6 months, working 12 hours daily, how many men would build 36 houses in 9 months, working 15 hours daily? *Ans.* 24 men.

46. If 15 men build 12 houses in 9 months, working 8 hours daily, how many such houses will 12 men build in 10 months, working 12 hours daily? *Ans.* 16 houses.

47. If 25 men build 36 houses in 9 months, working 12 hours daily, how many hours daily did 15 men work to build 24 houses in 12 months? *Ans.* 10 hrs. daily.

48. If 25 men build 15 houses in 9 months, working 9 hours daily, for 4 days in the week, in how many months will 15 men build 25 houses, working daily 12 hours, for 6 days in the week? *Ans.* 12½ mths.

49. If a journey be performed in 6 weeks, travelling at the rate of 9 miles per hour, for 12 hours daily, and for 5 days in the week, at what rate per hour must a person travel to perform a journey twice as long in 9 weeks, travelling 8 hours daily, for 6 days in the week? *Ans.* 15 mls. per hour.

50. Bought cloth for 14*s.* per yard, which I sold at

17*s.* 6*d.* per ell Flemish; what did I gain per cent., and how many yards did I sell to clear £60 13*s.* 4*d.*?

Ans. 33⅓ per cent.: sold 260 yds.

51. Bought cloth at 10*s.* 6*d.* per ell Flemish, and sold 120 yards at 23*s.* 4*d.* per yard; what did I gain by so doing, and what was the gain per cent.?

Ans. £56 gain, 66⅔ per cent.

52. Bought 3 tuns of oil for £157 10*s.*, and lost 56 gallons, at how much per gallon must I sell the remainder to retrieve the loss, and also clear £8 15*s.*?

Ans. 4*s.* 9*d.* per gall.

53. Bought 3 tuns of oil for £157 10*s.*, and after losing a certain number of gallons, sold the remainder at 4*s.* 9*d.* per gallon, by which I cleared £8 15*s.*; how many gallons did I lose? *Ans.* 56 galls.

54. If when wheat is 7*s.* 6*d.* per bushel, the penny loaf weigh 5 oz., what should it weigh when wheat is 6*s.* 3*d.* per bushel? *Ans.* 6 oz.

55. If when wheat is 5*s.* 7½*d.* per bushel, the penny loaf weigh 8 oz., what will be the price of wheat per bushel when it weighs only 6 oz.? *Ans.* 7*s.* 6*d.*

56. How many gallons of water must I add to a pipe of wine, which cost £91 10*s.*, that I may sell it at 12*s.* per gallon, without loss or gain? *Ans.* 26½ galls. water.

57. How many gallons of water must be added to a pipe of wine, value £91 10*s.*, that by selling it at 12*s.* per gallon, I may gain 10 per cent.? *Ans.* 41¾ galls. water.

58. What do I gain or lose, by selling at 12*s.* per gallon a pipe of wine which cost £86 8*s.*, and how much is the gain or loss per cent.?

Ans. £10 16*s.* loss, 12½ per cent.

59. If by selling wine at 12*s.* per gallon I lose 25 per cent., at what must I sell it per gallon to gain 25 per cent.?

Ans. 20*s.* per gall.

60. If wine, when pure, can be sold at 16*s.*, and when mixed with water at 12*s.* per gallon, how many gallons of water were added to the pipe? *Ans.* 42 galls. water.

61. If an ounce of silver be worth 5*s.* 6*d.*, what will be the cost of a service of plate weighing 324 oz. 15 dwts. 12 grs., allowing 1*s.* 10*d.* per oz. for fashion?

Ans. £119 1*s.* 8$\frac{1}{5}$*d.*

62. If a person travel 192 miles in 4 days, travelling 8 hours daily, in how many days will he travel 108 miles, travelling only 6 hours per day? *Ans.* 3 days.

63. If a journey can be performed in 6 days, travelling at the rate of 4 miles per hour, for 3 hours every day; at what rate per hour may a journey twice as long be performed in 3 days, travelling 4 hours daily? *Ans.* 12 mls. per hr.

64. If £150 worth of wine, at 24*s.* per gallon, will supply 12 men for 8 months; how many men may be supplied for 12 months with wine, at 16*s.* per gallon, for £250? *Ans.* 20 men.

65. If £150 worth of wine, at 24*s.* per gallon, will serve 12 persons for 8 months; for what sum of money can 20 persons be supplied for 12 months, with wine at 16*s.* per gallon? *Ans.* £250.

66. If £250 worth of wine, at 16*s.* per gallon, will serve 20 persons for 12 months; for how long may 12 persons be supplied with £150 worth of wine at 24*s.* per gallon?

Ans. 8 mths.

67. If £150 worth of wine, at 24*s.* per gallon, serve 12 persons for 8 months; what was the price of wine per gallon, when £250 worth supplied 20 persons for 12 months?

Ans. 16*s.* per. gall.

68. If £432 worth of wine, at 16*s.* per gallon, will serve 16 persons for 6 months, each drinking 1$\frac{1}{2}$ pints per day; how much may each person drink per day to make £324 worth, at 12*s.* per gallon, serve 9 persons for 8 months?

Ans. 2 pts.

69. A merchant barters 1300 reams of paper, worth 20*s.* per ream, for broad cloth at 26*s.* per yard; what number of yards must he receive? *Ans.* 1000 yds.

70. A merchant barters 1300 reams of paper, at 20*s.* per ream, for equal quantities of cambric at 13*s.*, and broad cloth at 26*s.* per yard; what quantity of each will he receive? *Ans.* 666$\frac{2}{3}$ yds. of each.

71. A merchant barters 1300 reams of paper, at 20*s*. per ream, for cambric at 13*s*., and cloth at 26*s*. per yard, but wishes to have twice as much cambric as cloth; how many yards of each will he receive?

Ans. 1000 yds. camb., 500 yds cloth.

72. A merchant barters 1300 reams of paper, at 20*s*. per ream, for Hollands at 26*s*., and brandy at 39*s*. per gallon; how many gallons of each will he receive?

Ans. 400 galls. of each.

73. A merchant barters 390 lb. of cinnamon, at 18*s*. per lb., for wool at 13*s*., and tea at 6*s*. 6*d*. per lb., but wishes to have twice as much tea as wool; how many lb. of each will he receive? *Ans.* 225 lb. wl., 450 lb. tea.

74. A merchant barters 23 cwt. 0 qrs. 24 lb. of tobacco, at 4*s*. 6*d*. per lb., for muslin at 4*s*. 4*d*., and velvet at 6*s*. 6*d*. per yard, wishing to have three times as much muslin as velvet; what quantity of each will he receive?

Ans. 1800 yds. mus., 600 yds. vel.

75. How many tierces of wine, at 15*s*. per gallon, must be given in barter for 12 hhds. of brandy at 35*s*. per gallon? *Ans.* 42 tierces.

76. A wall that was to be built to the height of 27 feet, was raised 9 feet by 12 men in 6 days; how many men must be employed to finish it in 4 days, working at the same rate? *Ans.* 36 men.

77. A person employed 36 men to build a wall 108 yards long in 20 days; but after they had worked 5 days, he altered the length of the wall to 72 yards, to be completed in 15 days from the commencement; how many men must be discharged? *Ans.* 6 men.

78. Bought 200 Flemish ells of cloth for £50, and 100 English ells of stuff for £20 16*s*. 8*d*.; find, without ascertaining the price of a yard of either, how many times the price of the cloth is greater or less than the price of the stuff per yard? *Ans.* twice as great.

79. Bought 150 lb. of tea for £56 5*s*., and 240 lb. of coffee for £30; find, without knowing the price of a lb. of either, how many times the price of the coffee was greater or less than the price of tea per lb.? *Ans.* 3 times less.

80. Bought 60 English ells of stuff at 3*s.* 4*d.* per yard, and 100 Flemish ells of cloth at 6*s.* 8*d.* per yard; find, without ascertaining either of the amounts, how many times the sum of money paid for stuff was greater or less than that paid for the cloth? *Ans.* twice as small.

81. Bought 225 lb. of coffee, at 2*s.* 6*d.* per lb., and 150 lb. of tea, at 7*s.* 6*d.* per lb.; find, without knowing the amount of either, how many times the sum of money paid for the tea was greater or less than that paid for the coffee? *Ans.* twice as great.

82. If 250 men dig a trench 450 yards long, 15 feet wide, and 5 feet deep in 21 days; how many men will dig a trench 360 yards long, 12 feet wide, and 3 feet deep in 14 days? *Ans.* 144 men.

83. If 125 men dig a trench 225 yards long, 15 feet wide, and 5 feet deep in 21 days; what must be the length of a trench 12 feet wide and 3 feet deep, dug by 72 men in 15 days? *Ans.* 180 yds.

84. If 175 men dig a trench 450 yards long, 10 feet wide, and 8 feet deep in 21 days, working 8 hours daily; how many men will dig a trench 360 yards long, 8 feet wide, and 6 feet deep in 14 days, working 12 hours daily; the ground of the latter being so much harder that 3 yards only can be dug in the same time as 5 of the former? *Ans.* 140 men.

85. If 350 men dig a trench 900 yards long, 16 feet wide, and 12 feet deep, in 25 days, working 12 hours daily; in how many days will 175 men dig a trench 720 yards long, 12 feet wide, and 8 feet deep, working 8 hours daily; the ground in the latter instance so much easier to work that 8 yards may be dug in the same time as 5 of the former, and the men so much more skilful as to do in 4 days as much as the others could in 5 days? *Ans.* 15 days.

86. If by selling a pipe of wine for £52 I lose 35 per cent., what was the prime cost of the pipe? *Ans.* £80.

87. If I lose 35 per cent. by selling a pipe of wine for £52, for what should I sell it to gain 25 per cent.? *Ans.* £100.

88. If the difference between selling a pipe of wine at 35 per cent. loss and 25 per cent. gain be £48, for how much was the pipe sold to lose 35 per cent.? *Ans*. £52.

89. If the difference between selling a pipe of wine at 15 per cent. loss and 20 per cent. gain be £28, what was the prime cost of the pipe? *Ans*. £80.

90. After losing 26 gallons from a pipe of wine, which cost £75, at what price per gallon must I sell the remainder, not only to retrieve the loss but also to gain 20 per cent. upon the prime cost? *Ans*. 18*s*. per gall.

91. After losing 26 gallons from a pipe of wine, and selling the remainder at 18*s*. per gallon, I gain 20 per cent. upon the prime cost, what was the prime cost of the pipe? *Ans*. £75.

92. If, after losing part of a pipe of wine, which cost £75, and selling the remainder at 18*s*. per gallon, I gain 20 per cent., how many gallons were lost? *Ans*. 26 galls.

93. If a garrison of 350 men, having provisions for 4 months, at the rate of 24 ounces daily per man, send away 50 of their men; to what must they reduce their daily allowance to enable them to hold out for 7 months? *Ans*. 16 oz.

94. A garrison of 3500 men, provisioned for 4 months at the rate of 24 ozs. per day, reduce their daily allowance to 16 ozs., and send away 50 of their men; how long will they be able to defend the post? *Ans*. 7 mths.

95. A garrison of 350, provisioned for 5 months, at the rate of 24 ozs. daily, receive a reinforcement of 150 men, and are compelled to hold out for 7 months; to what must they reduce their daily allowance? *Ans*. 12 ozs.

96. A garrison of 540 men, provisioned for 5 months at the rate of 24 ozs. per day, are compelled to hold out for 9 months; if they reduce their daily allowance to 18 ozs., how many of their men must they send away? *Ans*. 140 men.

97. If 3 lb. of tea be worth 7 lb. of coffee; 7 lb. of coffee worth 5 lb. of chocolate; 5 lb. of chocolate worth 20 lb. of honey; 20 lb of honey worth 30 lb. of soap; and 30 lb. of soap worth 50 lb. of rice; how much rice can I have for 15 lb. of tea? *Ans*. 250 lb. rice.

98. If 30 hhds. of brandy be worth 20 pipes of port wine; 25 pipes of port worth 24 pipes of claret; 20 pipes of claret worth 18 pipes of Madeira, and 6 pipes of Madeira worth 35 hhds. of rum; how many hhds. of rum can I have for 25 hhds. of brandy? *Ans.* 84 hhds. rum.

99. If 30 men will do a piece of work in a certain number of weeks, working a certain number of days each week, and a certain number of hours daily; how many men will do another piece of work 3 times as large, in twice the number of weeks, working half as many more days in the week, and for half the number of hours daily?

Ans. 60 men.

100. If 150 cubic yards of rock can be excavated in a certain number of days, by a certain number of men, working for a certain number of hours daily; how many cubic yards may be excavated in two-thirds of the number of days, by three times the number of men, working three-fourths the number of hours daily; supposing the rock in the latter instance to be so much harder that 3 yards only can be done in the same time as 5 of the former, and the men so much less skilful as to do only two-thirds as much work in the same time as the former?

Ans. 90 cub. yds.

COMMON MEASURE.

1. A NUMBER is said to be a measure of another number, when it is contained in it any number of times without a remainder: thus, 5 is a measure of 15, being contained in 15 three times without any remainder; for $15 \div 5 = 3$.

2. If a number measure another number, it will also measure any multiple of that number: thus, if 5 measure 15, it will also measure $15 \times 3 = 45$, or $15 \times 6 = 90$; for $45 \div 5 = 9$, and $90 \div 5 = 18$.

The reason of this is obvious; for it is evident that 5 must be contained just 3 times as often in 3 fifteens as in 1 fifteen, or 6 times as often in 6 fifteens as in 1 fifteen.

3. When a number is contained in each of two or more numbers without a remainder, it is called a common measure of those numbers; and the greatest number that is so contained in them, is called their greatest common measure.

Thus, 5 is a common measure of 30 and 45, being contained in each of those numbers without a remainder; but 15 is also contained in each of them without a remainder; and, as no number greater than 15 can be contained in each of them without a remainder, 15 is their greatest common measure.

4. If a number measure two other numbers, it will also measure the sum or the difference of those two numbers: thus, if 15 measure 30 and 45, it will also measure $30 + 45 = 75$, or $45 - 30 = 15$; for $75 \div 15 = 5$, and $15 \div 15 = 1$.

The reason of this is also obvious; for as 15 is contained in 30 twice, and in 45 three times, it must evidently be contained five times in both; and as 15 is contained just once oftener in 45 than in 30, it is equally evident that it must be contained just once in the difference of those two numbers.

Hence, to find the greatest common measure of two or more numbers, we have the following

RULE.

1. When there are only two numbers, divide the greater number by the smaller; and if there be no remainder, the divisor is their greatest common measure.

2. If there be a remainder, divide the divisor by this remainder, and continue to divide every succeeding divisor by the succeeding remainder till nothing remains; and the last divisor will be the greatest common measure required.

3. If there be more than two numbers, find the greatest common measure of any two of them, as before; and afterwards find the common measure of this common measure and any of the remaining numbers.

4. Continue this process, always taking the common measure last found and any of the remaining numbers, till all have been used; and the last common measure thus found will be the greatest common measure of all the numbers.

Ex. 1.—To find the greatest common measure of 15 and 45.

Here, dividing the greater of the two numbers, 45, by the smaller, 15, we have no remainder; therefore the divisor, 15, is the greatest common measure of the numbers 15 and 45.

```
15)45(3
   45
   --
```

Ex. 2.—To find the greatest common measure of 54 and 99.

Here, dividing the greater number, 99, by the smaller, 54, we have the remainder 45; and dividing the divisor 54 by this remainder 45, we get the remainder 9; and now, dividing the last divisor, 45, by this remainder 9, we have no remainder: consequently this last divisor, 9, is the greatest common measure of the two numbers 54 and 99.

```
54)99(1
   54
   --
   45)54(1
      45
      --
       9)45(5
         45
         --
```

1. That 9 is a common measure of 54 and 99 may be thus demonstrated: as 9 evidently measures itself, it must also measure any multiple of itself, and therefore measures $9 \times 5 = 45$; and since it measures itself and 45, it must measure the sum of these, $9 + 45 = 54$, which is one of the given numbers; again, since 9 measures 45 and 54, it must also measure the sum of these, $45 + 54 = 99$, which is the other of the given numbers: therefore 9 is a common measure of the numbers 54 and 99.

2. That 9 is the greatest common measure of these numbers may also be thus demonstrated: as 9 measures 54 and 99, it must also measure the difference of these numbers, $99 - 54 = 45$; and since 9 measures 54 and 45, it must also measure the difference of these numbers, $54 - 45 = 9$; now it is evident that no number greater than 9 can measure 9; therefore 9 is the greatest common measure of the numbers 54 and 99.

Ex. 3.—Find the greatest common measure of 36, 48, and 92.

Here we first find the greatest common measure of the two numbers 36 and 48, which is 12, and now proceed to find the greatest common measure of this common measure 12, and the remaining number 92, which is 4; consequently this last common measure, 4, is the greatest common measure of the three numbers 36, 48, and 92; for $36 \div 4 = 9$; $48 \div 4 = 12$; and $92 \div 4 = 23$.

```
36)48(1
   36
   --
   12)36(3
      36
      --
12)92(7
   84
   --
    8)12(1
       8
       -
       4)8(2
         8
```

EXAMPLES FOR PRACTICE.

1. Find the greatest common measure of the numbers 126 and 210. *Ans.* 42.

2. Of 540 and 612. *Ans.* 36.
3. Of 567 and 945; and 756 and 1092. *Ans.* 189, and 84.
4. Of the numbers 54, 99, and 117. *Ans.* 9.
5. Of 1134 and 3024. *Ans.* 378.
6. Of 3768 and 17976. *Ans.* 24.
7. Of the numbers 126, 210, and 315. *Ans.* 21.
8. Of 6048 and 22176. *Ans.* 2016.
9. Of the numbers 540, 612, and 846. *Ans.* 18.
10. Of the numbers 945, 1890, 2142, and 3213. *Ans.* 63.

COMMON MULTIPLE.

1. If two or more numbers be multiplied together, the product is called a common multiple of those numbers; and in this common multiple each of those numbers will be contained a certain number of times exactly, without any remainder.

Thus, if the numbers 3, 6, and 18 be multiplied together, $3 \times 6 \times 18 = 324$ is their common multiple: in which 3 is contained 6×18 times, or 108 times exactly; 6 is contained 3×18 times, or 54 times; and 18 is contained 3×6 times, or 18 times exactly, without any remainder.

2. But it may be possible to find a number smaller than 324, in which each of the numbers 3, 6, and 18 may be contained without remainder; and the smallest number in which they can be so contained is called their least common multiple.

Thus, each of the numbers 3, 6, and 18 is contained without remainder in the number 18; and as there is no number smaller than 18 in which 18, one of the given numbers, can be so contained, 18 is their least common multiple.

3. To find the least common multiple, there are several methods; of which the first and most obvious is, to strike out of the series of given numbers, each of those of which a multiple can be found in any of the other numbers.

Thus, in the series of given numbers 3, 6, and 18, a multiple of 3 is found in the number 6, which contains 3 twice; also a multiple of 6 is found in the number 18, which contains 6 three times: consequently we strike out from the series the numbers 3 and 6, and the remaining number, 18, is their least common multiple.

This will be obvious; for as 3 is contained in 6, it must be also contained in 18, which is a multiple of 6; and for the same reason 6 is also contained in 18; and, as 18 is evidently contained in itself, it is evident that each of the given numbers 3, 6, and 18 is contained in 18.

Ex.—To find the least common multiple of the numbers 3, 6, and 18.

$\cancel{3} \cdot \cancel{6} \cdot 18$ L. C. M.

Ex. 2.—To find the least common multiple of the given numbers 2, 4, 8, 16, 32, 64, and 128.

$\cancel{2} \cdot \cancel{4} \cdot \cancel{8} \cdot \cancel{16} \cdot \cancel{32} \cdot \cancel{64} \cdot 128$

Here 2 has a multiple in 4; 4 has a multiple in 8; 8 has a multiple in 16; 16 in 32; 32 in 64; and 64 in 128: consequently we strike out from the series the numbers 2, 4, 8, 16, 32, and 64; and the remaining number, 128, is the least common multiple of all the given numbers.

4. Another method of finding the least common multiple of several numbers, is to divide any two or more of them by their greatest common measure; and the continued product of the common measure and the several quotients, will be the least common multiple required.

Thus, to find the least common multiple of the numbers 8 and 12; dividing both these numbers by their greatest common measure, 4, we get the quotients 2 and 3; and multiplying these quotients together with the common measure 4, we have $4 \times 2 \times 3 = 24$, the least common multiple required.

5. If there are many numbers we have only to continue this process till there are no two numbers left that have a common measure; and multiplying together the common measures and the undivided numbers (if any), we shall obtain the least common multiple required.

Thus, to find the least common multiple of the numbers 8, 12, 14, and 21; dividing the numbers 8 and 12 by their common measure, 4, we get the quotients 2 and 3; and bringing down in a line with these the undivided numbers 14 and 21, we have in the second line the numbers 2, 3, 14, and 21; but of these, 2 has a multiple in 14, and 3 has a multiple in 21; we therefore strike out the numbers 2 and 3, and dividing 14 and 21 by their common measure, 7, we get the quotients 2 and 3; and as these have no common divisor, we multiply together the divisors 4 and 7 and the undivided numbers 2 and 3, and their product, $7 \times 4 \times 2 \times 3 = 168$, is the least common multiple required.

$$
\begin{array}{r|l}
4 & 8 \,.\, 12 \,.\, 14 \,.\, 21 \\
\hline
7 & \cancel{2} \,.\, \cancel{3} \,.\, 14 \,.\, 21 \\
\hline
 & 2 \,.\, 3
\end{array}
$$

$4 \times 7 \times 2 \times 3 = 168$ L. C. M.

Here it may be observed that the common multiple of the given numbers is $8 \times 12 \times 14 \times 21 = 28224$, and the least common multiple of the same numbers is $4 \times 7 \times 2 \times 3 = 168$ only; but in this least common multiple each of the given numbers 8, 12, 14, and 21 is contained without any remainder.

For as $4 \times 7 \times 2 \times 3$ is contained in 168, it is evident that $4 \times 2 = 8$, $4 \times 3 = 12$, $7 \times 2 = 14$, and $7 \times 3 = 21$, are also contained in 168 exactly, without any remainder; and consequently each of the given numbers is contained in 168, which is therefore their least common multiple.

Hence, to find the least common multiple of several numbers, we have the following

RULE.

1. Write down the given numbers in a line, and strike out of the series every number of which a multiple can be found in any of the other numbers; and if there be only one number left, that number will be the least common multiple required.

2. If there be several numbers left, divide any two or more of them by any number that will divide two of them at least without a remainder, and write

down the quotients, and also the undivided numbers, in a line below them.

3. Divide these quotients, and also the undivided numbers, by any number that will divide two of them at least, as before; and continue this process till there are no two numbers left which can be divided by the same number.

4. Lastly, multiply together all the divisors, the several quotients, and undivided numbers in the last line; and their continued product will be the least common multiple of all the given numbers.

EXAMPLES FOR PRACTICE.

1. Find the least common multiple of the numbers 3, 4, 6, and 8. *Ans.* 24.

2. Of the numbers 3, 4, 5, and 6. *Ans.* 60.

3. Of the numbers 2, 3, 5, and 8. *Ans.* 120.

4. Of the numbers 7, 14, 28, 56, and 112. *Ans.* 112.

5. Of the numbers 15, 27, 90, 108, and 360. *Ans.* 1080.

6. Of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9. *Ans.* 2520.

7. Of the numbers 2, 4, 6, 8, 10, 12, and 16. *Ans.* 240.

8. Of 128, 384, 768, and 2304. *Ans.* 2304.

9. Of $1\frac{1}{2}$, 7, 3, $3\frac{1}{2}$, $8\frac{1}{2}$, and 17. *Ans.* 357.

10. Of 3, 9, 27, 81, 243, and 729. *Ans.* 729.

FRACTIONS.

1. A FRACTION is a part or parts of a unit; and as we have no numbers less than 1 to express quantities which are less than a unit, the magnitude of a fractional quantity is expressed by two numbers, of which the one has to the other the same ratio as the part or parts contained in the fraction, have to the whole unit or integer.

2. Of these two numbers, one is written above the line which separates them, and is called the numerator, because

it shows how many parts of the unit or integer are contained in the fraction; and the other is written below the line, and is called the denominator, because it tells the name of the parts, by showing into how many such parts the whole unit or integer is divided.

Thus, if the unit 1, representing one entire thing of any kind, be divided into eight equal parts, and one of these parts be taken as the fraction; this fraction will be expressed by the numerator 1, showing that there is one of these parts contained in the fraction; and by the denominator 8, showing that the whole unit is divided into eight equal parts, and, consequently, that each of these parts is one-eighth, written $\frac{1}{8}$.

Or if five of these parts be taken as the fraction, this fraction will be expressed by the numerator 5, and the denominator 8, written $\frac{5}{8}$; showing that this fraction contains five of the eight parts into which the whole unit or integer is divided, and is consequently equal to five-eighths of the integer.

3. Hence the quantities $\frac{1}{8}$ and $\frac{5}{8}$ are fractions, and their magnitudes are expressed by the ratios $1 : 8$ and $5 : 8$ respectively, or the ratios which their numerators severally bear to their respective denominators, and which are also the ratios that the fractions $\frac{1}{8}$ and $\frac{5}{8}$ severally bear to the whole unit or integer.

4. From this view of the subject, it is obvious that the numerator of a fraction is the antecedent, and the denominator the consequent of the ratio which the fraction bears to its integer; and consequently that the doctrine of fractions and of ratio is the same.

5. If the numerator of a fraction be increased, and the denominator remain unaltered, the magnitude of the fraction will be increased; or if the numerator be decreased, the magnitude of the fraction will be decreased.

For the numerator shows how many parts of the integer are contained in the fraction; and it is obvious that if the number of these parts be increased, the fraction will be

increased; or if the number of the parts be decreased, the fraction will be decreased.

Thus, let the numerator of the fraction $\frac{3}{4}$ be made twice as great, then $\frac{3\times2}{4}=\frac{6}{4}$, which last fraction evidently contains just twice as many parts of the same magnitude, as the fraction $\frac{3}{4}$, and is consequently just twice as great.

Or let the numerator of the fraction $\frac{6}{4}$ be made twice as small, then $\frac{6\div2}{4}=\frac{3}{4}$, which last fraction evidently contains just half as many parts of the same magnitude, as the fraction $\frac{6}{4}$, and is consequently just twice as small.

Hence, it is evident that fractions vary directly as their numerators.

6. If the denominator of a fraction be increased and the numerator remain unaltered, the magnitude of the fraction will be decreased; or if the denominator be decreased, the magnitude of the fraction will be increased.

For, the denominator shows into how many parts the integer is divided; and it is obvious that the greater the number of the parts into which the integer is divided, the smaller must be the magnitude; and the smaller the number of the parts, the greater must be the magnitude of each part.

Thus, let the denominator of the fraction $\frac{3}{4}$ be made twice as great, then $\frac{3}{4\times2}=\frac{3}{8}$, which last fraction evidently contains the same number of parts, each of which is just twice as small as the parts of the fraction $\frac{3}{4}$, and is consequently just twice as small.

Or let the denominator of the fraction $\frac{3}{8}$ be made twice as small, then $\frac{3}{8\div2}=\frac{3}{4}$, which last fraction evidently contains the same number of parts, each of which is just twice as great as the parts of the fraction $\frac{3}{8}$, and is consequently just twice as great.

Hence, it is evident that fractions vary inversely as their denominators.

7. If the integer of a fraction be increased, and the value remain unaltered, the magnitude of the fraction must be decreased; or if the integer be decreased, the magnitude of the fraction must be increased.

For it is evident that any part of a greater integer must be greater than the same part of a smaller; and also that any part of a smaller integer must be less than the same part of a greater.

Consequently, that the original value of the fraction may remain unaltered, it is evident that the fraction must be made as many times less, as the integer is made greater; or as many times greater, as the integer is made less.

Thus, to express $\frac{3}{4}$ of a shilling as an equivalent fraction of a £.; as the integer £1 is 20 times as great as the integer 1*s*., the fraction $\frac{3}{4}$ must be made 20 times as small, and we shall have $\frac{3}{4 \times 20} = \frac{3}{80}$ £.

Also, to express $\frac{3}{80}$ of a £. as an equivalent fraction of a shilling; as the integer 1*s*., is 20 times as small as the integer £1, the fraction $\frac{3}{80}$ must be made 20 times as great, and we shall have £. $\frac{3 \times 20}{80} = \frac{60}{80} = \frac{3}{4}$ *s*.

Hence it is evident that equivalent fractions vary inversely as their integers.

From these obvious principles, are derived all the rules for the solution of problems in fractional quantities.

VULGAR FRACTIONS.

A "vulgar fraction" is the common appellation of all such fractions as ordinarily occur; without restriction to any particular number of parts into which the integer may be divided, from which circumstance some of them derive particular names.

Thus, a decimal fraction, is a fraction of which the integer is divided into ten equal parts; a duodecimal fraction, is a fraction of which the integer is divided into twelve

equal parts, &c., &c.; but a vulgar fraction, is a fraction of which the integer may be divided into any number of parts whatever.

Vulgar fractions are either Proper, Improper, Mixed, Complex, Simple, or Compound.

1. A Proper fraction, is a fraction in which the number of parts taken, is less than the number of parts into which the integer is divided; and, consequently, of which the numerator is less than the denominator.

Thus, $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}$, and $\frac{7}{8}$, are all proper fractions; but if we take one part more, the quantity, $\frac{8}{8}$, will be equal to the whole integer, and therefore cannot properly be called a fraction.

2. An Improper fraction, is a fraction in which the number of parts taken, is either equal to, or greater than, the number of parts into which the integer is divided; and consequently, of which the numerator is either equal to, or greater than, the denominator.

Thus $\frac{8}{8}$ and $\frac{11}{8}$ are improper fractions, because the number of parts in the first $\frac{8}{8}$, is equal to, and in the second $\frac{11}{8}$, greater than, the number of parts into which the integer is divided; the latter containing not only the whole integer $\frac{8}{8}$, but also $\frac{3}{8}$ of another integer, being equal to 1 and $\frac{3}{8}$ or $1\frac{3}{8}$, which is a mixed number.

But as it may be convenient to express either whole or mixed numbers in the form of a fraction, such quantities as $\frac{8}{8}$ and $\frac{11}{8}$ are called fractions, but distinguished from quantities which are really such, by the appellation Improper fractions.

3. A Mixed number is a quantity consisting of a whole number, and also a fraction of another unit, as $1\frac{3}{8}$, which is equal to the improper fraction $\frac{11}{8}$, the latter being only a more convenient fractional expression for that quantity.

4. A Complex fraction is a fraction of which either the numerator or the denominator is itself a fraction, or a mixed number; or of which both the numerator and the denominator are themselves fractions or mixed numbers.

Thus,

$$\frac{\frac{3}{5}}{9},\quad \frac{7}{\frac{3}{5}},\quad \frac{3\frac{1}{2}}{5},\quad \frac{5}{7\frac{1}{3}},\quad \frac{\frac{4}{5}}{\frac{7}{8}},\quad \text{and}\quad \frac{7\frac{1}{2}}{9\frac{2}{3}}$$

are all Complex fractions.

5. A Simple fraction, is a fraction in which the parts taken are parts of a unit; and consequently bear to the whole unit or integer, the simple ratio which the numerator of the fraction bears to its denominator.

Thus, $\frac{5}{6}$ is a simple fraction, because the sixth parts which it contains are sixth parts of the unit or integer; and consequently bear to the whole unit, the simple ratio which the numerator of the fraction bears to its denominator, $5 : 6 = \frac{5}{6}$.

6. A Compound fraction, is a fraction in which the parts taken are not parts of a unit or of the integer, but of some parts into which the integer may have been previously divided; and, consequently, of which the magnitude will be expressed by the ratio compounded of the various ratios, which its several numerators bear to their respective denominators.

Thus $\frac{3}{4}$ of $\frac{5}{6}$ is a compound fraction; because the fourth parts which it contains, are not fourth parts of the integer, but of the sixth parts into which the integer has been previously divided; and of which the magnitude is expressed by the ratio compounded of the ratios $3 : 4$ and $5 : 6$, or $\overline{3 \times 5} : \overline{4 \times 6} = 15 : 24 = \frac{15}{24}$.

REDUCTION.

1. Reduction of vulgar fractions is the changing of their form or denomination, without altering their value;

and as the numerator and denominator of a fraction are respectively the antecedent and consequent of the ratio which the fraction bears to its integer, we have only to apply the general properties of ratio to this purpose.

Thus, if the antecedent and consequent of a ratio be both multiplied, or both divided by the same number, the ratio is not altered (See Ratio, Arts. 9 and 10); consequently, if the numerator and denominator of a vulgar fraction, be both multiplied or both divided by the same number, the magnitude of the fraction is not altered.

From this property of ratio, we obtain a general rule for the solution of all the problems in reduction of vulgar fractions, with the exception of two only. These exceptions are,

1. The reduction of a compound fraction to a simple fraction, which is nothing more than the composition of the several ratios which the fractions respectively bear to their integers (see Ratio, Art. 11); and

2. The reduction of a fraction having any given integer, to an equivalent fraction having any other integer required; for which a rule is obtained from the principle, that equivalent fractions vary inversely as their integers.

PROBLEM I.

To reduce a whole number to an equivalent vulgar fraction, having any denominator required.

Every whole number is equal to a fraction having that number for its numerator, and the unit 1 for its denominator; thus, $9 = \frac{9}{1}$; and if we multiply both terms of this fraction by the required denominator, we shall have the equivalent fraction required.

Thus, to reduce the whole number 9 into an equivalent vulgar fraction, having the denominator 12; first $9 = \frac{9}{1}$, and $\frac{9}{1} = \frac{9 \times 12}{1 \times 12} = \frac{108}{12}$, the equivalent fraction required. Hence the

RULE.

Make the given whole number the numerator of a fraction, having 1 for its denominator; and multiply both terms of this fraction by the denominator of the fraction required.

EXAMPLES FOR PRACTICE.

1. Reduce the whole number 7 to a vulgar fraction, having the denominator 12. *Ans.* $\frac{84}{12}$.

2. Reduce 17 to a fraction, having the denominator 17. *Ans.* $\frac{289}{17}$.

3. Reduce 25 to a fraction, having the denominator 30. *Ans.* $\frac{750}{30}$.

4. Reduce 9 to a fraction, having the denominator 45. *Ans.* $\frac{405}{45}$.

5. Reduce 100 to a fraction, having the denominator 100. *Ans.* $\frac{10000}{100}$.

6. Reduce 252 to a fraction, having the denominator 125. *Ans.* $\frac{31500}{125}$.

PROBLEM II.

To reduce a vulgar fraction into its lowest terms.

RULE.

1. Divide both terms of the given fraction, and also the resulting quotients, by any number that will divide them both without a remainder. Or, 2. Divide both terms of the given fraction, by their greatest common measure.

Ex.—To reduce the vulgar fraction $\frac{576}{768}$ into its lowest terms.

1. $\frac{567}{768} = \frac{576 \div 12}{768 \div 12} = \frac{48}{64} = \frac{48 \div 8}{64 \div 8} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$, lowest term.

2. Or, finding the greatest common measure of the numbers 576 and 768 to be 192, we divide both terms of the given fraction by this common measure 192,

$$\begin{array}{l} 576\,)\,768 \\ \quad\;\; \underline{576} \\ \quad\;\; 192\,)\,576\,(\,3 \\ \qquad\quad\;\; \underline{576} \\ \qquad\quad\;\; \cdots \end{array}$$

$$\frac{576}{768} = \frac{576 \div 192}{768 \div 192} = \frac{3}{4}, \text{ lowest term.}$$

and obtain the quotient of the numerator $=3$, and the quotient of the denominator $=4$, which are the numerator and denominator of the given fraction in its lowest terms, $\frac{3}{4}$.

EXAMPLES FOR PRACTICE.

1. Reduce the vulgar fraction $\frac{540}{612}$ into its lowest terms. *Ans.* $\frac{15}{17}$.

2. Reduce $\frac{976}{1584}$ to its lowest terms. *Ans.* $\frac{61}{99}$.

3. Reduce $\frac{1134}{3024}$ to its lowest terms. *Ans.* $\frac{3}{8}$.

4. Reduce $\frac{4512}{14256}$ to its lowest terms. *Ans.* $\frac{94}{297}$.

5. Reduce $\frac{18900}{35560}$ to its lowest terms. *Ans.* $\frac{135}{254}$.

6. Reduce $\frac{758016}{2395008}$ to its lowest terms. *Ans.* $\frac{94}{297}$.

PROBLEM III.

To reduce a mixed number to an equivalent improper fraction.

RULE.

Make the given mixed number the numerator of a fraction having 1 for its denominator; and multiply both terms of this fraction by the denominator of the fractional part of the mixed number.

Ex.—To reduce the mixed number $12\frac{5}{7}$ to an equivalent improper fraction.

$$12\tfrac{5}{7} = \frac{12\frac{5}{7}}{1}, \text{ and } \frac{12\frac{5}{7}}{1} = \frac{12\frac{5}{7} \times 7}{1 \times 7} = \frac{89}{7}, \text{ the improper fraction.}$$

Or, for practical use, multiply the whole number by the denominator of the fractional part of the mixed number, and add the numerator to the product, writing the denominator under their sum, thus $12\frac{5}{7} = \frac{\overline{12 \times 7 + 5}}{7} = \frac{89}{7}$.

EXAMPLES FOR PRACTICE.

1. Reduce the mixed number $5\frac{3}{7}$ to an equivalent improper fraction. *Ans.* $\frac{38}{7}$.

2. Reduce $9\frac{3}{11}$ and $12\frac{5}{12}$ to improper fractions. *Ans.* $\frac{102}{11}$ and $\frac{149}{12}$.

3. Reduce $17\frac{7}{8}$ and $18\frac{8}{9}$ to improper fractions. *Ans.* $\frac{143}{8}$ and $\frac{170}{9}$.

4. Reduce $25\frac{9}{10}$ and $30\frac{13}{15}$ to improper fractions. *Ans.* $\frac{259}{10}$ and $\frac{463}{15}$.

5. Reduce $125\frac{109}{120}$ and $37\frac{19}{30}$ to improper fractions. *Ans.* $\frac{15109}{120}$ and $\frac{1129}{30}$.

6. Reduce $120\frac{13}{96}$ and $90\frac{117}{128}$ to improper fractions. *Ans.* $\frac{11533}{96}$ and $\frac{11637}{128}$.

PROBLEM IV.

To reduce an improper fraction to an equivalent whole or mixed number.

RULE.

Divide both terms of the improper fraction by its denominator, or divide the numerator by the denominator; and if there be no remainder the quotient will be the whole number, or if there be a remainder, the mixed number required.

Ex. 1.—To reduce $\frac{84}{7}$ to an equivalent whole or mixed number. $\frac{84}{7} = \frac{84 \div 7}{7 \div 7} = \frac{12}{1} = 12$.

Ex. 2 —To reduce $\frac{89}{7}$ to an equivalent whole or mixed number. $\frac{89}{7} = \frac{89 \div 7}{7 \div 7} = \frac{12\frac{5}{7}}{1} = 12\frac{5}{7}$.

EXAMPLES FOR PRACTICE.

1. Reduce the improper fraction $\frac{132}{11}$, to an equivalent whole or mixed number. *Ans.* 12.

2. Reduce $\frac{150}{12}$ to a whole or mixed number. *Ans.* $12\frac{1}{2}$.

3. Reduce $\frac{125}{15}$ and $\frac{120}{21}$ to whole or mixed numbers. *Ans.* $8\frac{1}{3}$ and $5\frac{5}{7}$.

4. Reduce $\frac{5275}{25}$ and $\frac{1234}{96}$ to whole or mixed numbers. *Ans.* 211 and $12\frac{41}{48}$.

5. Reduce $\frac{3456}{156}$ and $\frac{4567}{234}$ to whole or mixed numbers. *Ans.* $22\frac{2}{13}$ and $19\frac{121}{234}$.

6. Reduce $\frac{70956}{729}$ and $\frac{170356}{936}$ to whole or mixed numbers. *Ans.* $97\frac{1}{3}$ and $182\frac{1}{234}$.

PROBLEM V.

To reduce a complex fraction to an equivalent simple fraction.

RULE.

If the fraction be complex in one of its terms only, multiply both terms of the fraction by the denominator of the fraction in the complex term; or if it be complex in both terms, multiply both the terms by the least common multiple of the denominators of the fractions in those terms.

Ex. 1.—To reduce the complex fraction $\frac{\frac{3}{5}}{7}$ to an equivalent simple fraction. $\frac{\frac{3}{5}}{7} = \frac{\frac{3}{5} \times 5}{7 \times 5} = \frac{3}{35}$.

Ex. 2.—To reduce the complex fraction $\frac{5}{7\frac{3}{5}}$ to an equivalent simple fraction. $\frac{5}{7} = \frac{5 \times 5}{7\frac{3}{5} \times 5} = \frac{25}{38}$.

Ex. 3.—To reduce the complex fraction $\dfrac{\frac{7}{8}}{\frac{11}{12}}$ to an equivalent simple fraction.

Here the least common multiple of the denominators 8 and 12 is 24, and $\dfrac{\frac{7}{8}}{\frac{11}{12}} = \dfrac{\frac{7}{8}\times 24}{\frac{11}{12}\times 24} = \dfrac{21}{22}$.

Ex. 4.—To reduce the complex fraction $\dfrac{4\frac{5}{6}}{5\frac{7}{9}}$ to an equivalent simple fraction.

Here the least common multiple of the denominators 6 and 9 is 18, and $\dfrac{4\frac{5}{6}}{5\frac{7}{9}} = \dfrac{4\frac{5}{6}\times 18}{5\frac{7}{9}\times 18} = \dfrac{87}{104}$.

EXAMPLES FOR PRACTICE.

1. Reduce the complex fraction $\dfrac{2\frac{1}{2}}{3}$ to an equivalent simple fraction. *Ans.* $\dfrac{5}{6}$.

2. Reduce $\dfrac{4}{5\frac{1}{3}}$ and $\dfrac{5\frac{1}{4}}{7}$ to simple fractions. *Ans.* $\dfrac{12}{16} = \dfrac{3}{4}$, and $\dfrac{21}{28}$.

3. Reduce $\dfrac{3\frac{3}{4}}{7\frac{1}{2}}$ and $\dfrac{7\frac{1}{5}}{9\frac{3}{10}}$ to simple fractions. *Ans.* $\dfrac{15}{30} = \dfrac{1}{2}$, and $\dfrac{72}{93}$.

4. Reduce $\dfrac{\frac{5}{17}}{\frac{23}{34}}$ and $\dfrac{35\frac{1}{3}}{176\frac{2}{3}}$ to simple fractions. *Ans.* $\dfrac{10}{23}$ and $\dfrac{1}{5}$.

5. Reduce $\dfrac{23\frac{8}{21}}{163\frac{2}{3}}$ and $\dfrac{\frac{15}{90}}{\frac{35}{70}}$ to simple fractions. *Ans.* $\dfrac{1}{7}$ and $\dfrac{1}{3}$.

6. Reduce $\dfrac{\frac{27}{81}}{\frac{123}{162}}$ and $\dfrac{35\frac{5}{7}}{500}$ to simple fractions. *ns.* $\dfrac{18}{41}$ and $\dfrac{1}{14}$.

PROBLEM VI.

To reduce a compound fraction to an equivalent simple fraction.

Note.—A compound fraction bears to the integer a ratio compounded of all the ratios which its several numerators bear to their respective denominators (see Ratio, Art. 11): hence the

RULE.

Multiply all the numerators together, and also all the denominators; and the product of the numerators will be the numerator, and the product of the denominators will be the denominator of the simple fraction.

Ex.—To reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ to a simple fraction equivalent.

$$\frac{3}{4} \text{ of } \frac{5}{6} \text{ of } \frac{7}{8} = \frac{\cancel{3}\times5\times7}{4\times\underset{2}{\cancel{6}}\times8} = \frac{35}{64}.$$

Ex. 2.—To reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$ to an equivalent simple fraction.

$$\frac{3}{4} \text{ of } \frac{4}{5} \text{ of } \frac{5}{8} = \frac{3\times\cancel{4}\times\cancel{5}}{\cancel{4}\times\cancel{5}\times8} = \frac{3}{8}.$$ (See Ratio, Art. 11.)

EXAMPLES FOR PRACTICE.

1. Reduce the compound fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a simple fraction. *Ans.* $\frac{1}{3}$.

2. Reduce $\frac{5}{6}$ of $\frac{7}{9}$ of $\frac{3}{10}$, and $\frac{2}{15}$ of $4\frac{1}{2}$ to simple fractions. *Ans.* $\frac{7}{36}$ and $\frac{3}{5}$.

3. Reduce $\frac{11}{21}$ of $\frac{14}{88}$, and $\frac{3}{17}$ of $\frac{4\frac{6}{7}}{9}$ to simple fractions. *Ans.* $\frac{1}{12}$ and $\frac{2}{21}$.

4. Reduce $\frac{3}{5}$ of $\frac{6}{17}$ of $\frac{7}{9}$ of $\frac{11}{42}$ of 15 to a simple fraction. *Ans.* $\frac{11}{17}$.

5. Reduce $\frac{2}{9}$ of $\frac{17}{36}$ of $\frac{63}{85}$, and $\frac{75}{112}$ of $\frac{28}{37\frac{1}{2}}$ to simple fractions. *Ans.* $\frac{7}{90}$ and $\frac{1}{2}$.

6. Reduce $\frac{35\frac{1}{3}}{176\frac{2}{3}}$ of $\frac{27\frac{3}{11}}{109\frac{4}{44}}$ of $\frac{1}{11}$ of $\frac{132}{12}$ to a simple fraction. *Ans.* $\frac{1}{20}$.

PROBLEM VII.

To reduce several fractions having different denominators to equivalent fractions, all having the same or one common denominator.

RULE.

Take the least common multiple of all the denominators for the common denominator of all the given fractions; and multiply both terms of each fraction, by such a number as will make its denominator equal to the common denominator. *Note.*—This multiplier may be found by dividing the common denominator by the denominator of the fraction.

Note.—The rule generally given for the solution of this problem is to multiply the numerator of each of the fractions by all the denominators, except its own, for a new numerator, and to multiply all the denominators together for a common denominator. Thus, to reduce the fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ to equivalent fractions having a common denominator.

$$\begin{array}{l} 3\times6\times8=144 \\ 5\times4\times8=160 \\ 7\times4\times6=168 \\ \hline 4\times6\times8=192 \end{array}$$

If we look into this process, we shall find that the numerator 3 of the fraction $\frac{3}{4}$ is multiplied by the denominators 6 and 8; and in the line of denominators we find its denominator 4 also multiplied by the same numbers 6 and 8; consequently the resulting fraction $\frac{144}{192}$ is equal to the given fraction $\frac{3}{4}$, and so of all the other fractions: therefore the rule is true. But it is objectionable, first, as not bringing out the required fractions in their lowest terms; secondly, as not exhibiting the fractions distinctly; and thirdly, because the intricacy of the arrangement conceals the principle on which the rule is

founded. On the contrary, the rule given above always brings out the required fractions in the lowest terms in which they can be expressed, subject to the condition of their all having a common denominator; and from the simplicity of the arrangement, while it distinctly exhibits each of the required fractions, it clearly demonstrates the principle on which the process is founded.

Ex.—To reduce the fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ to equivalent fractions having all one common denominator.

Here the least common multiple of the denominators 4, 6, and 8, is 24, and multiplying both terms of the fraction $\frac{3}{4}$ by $24 \div 4 = 6$, we get the equivalent fraction $\frac{18}{24}$; also multiplying both terms of the fraction $\frac{5}{6}$ by $24 \div 6 = 4$, we get the equivalent fraction $\frac{20}{24}$; and, lastly, multiplying both terms of the fraction $\frac{7}{8}$ by $24 \div 8 = 3$, we get the equivalent fraction $\frac{21}{24}$; all which fractions are respectively equal to the given fractions, and have the common denominator 24.

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$
$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$
$$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

EXAMPLES FOR PRACTICE.

1. Reduce the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to equivalent fractions, all having one common denominator.

Ans. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

2. Reduce $\frac{3}{4}$, $\frac{5}{8}$, $\frac{9}{10}$, and $\frac{11}{16}$ to a common denominator.

Ans. $\frac{60}{80}$, $\frac{50}{80}$, $\frac{72}{80}$, and $\frac{55}{80}$.

3. Reduce $\frac{5}{8}$, $\frac{9}{16}$, $\frac{11}{32}$, and $\frac{15}{64}$ to a common denominator.

Ans. $\frac{40}{64}$, $\frac{36}{64}$, $\frac{22}{64}$, and $\frac{15}{64}$.

4. Reduce $\frac{7}{15}$ of $\frac{5}{8}$, $\frac{2}{3}$ of $\frac{11}{16}$, and $2\frac{4}{15}$, to a common denominator.

Ans. $\frac{35}{120}$, $\frac{55}{120}$, and $\frac{272}{120}$.

5. Reduce $\frac{15}{16}$, $\frac{23}{24}$, $\frac{11}{12}$, and $\frac{37}{48}$ to a common denominator.

Ans. $\frac{45}{48}$, $\frac{46}{48}$, $\frac{44}{48}$, and $\frac{37}{48}$.

6. Reduce $\frac{2}{3}$ of $\frac{7}{8}$, $\frac{5}{7}$ of $\frac{11}{12}$, and $\frac{3}{4}$ of $\frac{19}{21}$, to a common denominator.

Ans. $\frac{49}{84}$, $\frac{55}{84}$, and $\frac{57}{84}$.

7. Reduce $\frac{3}{4}$ of $\frac{9\frac{1}{2}}{13\frac{1}{2}}$ and $\frac{7}{17}$ of $\frac{19}{21}$ of $\frac{4\frac{1}{4}}{18}$ to a common denominator.

Ans. $\frac{114}{216}$ and $\frac{19}{216}$.

8. Reduce $\frac{9}{11}$ of $\frac{44}{63}$ of $\frac{4\frac{3}{8}}{9}$, $\frac{13}{24}$ of $\frac{125}{156}$ of $\frac{7}{9\frac{3}{8}}$, and $\frac{14}{15}$ of $\frac{25}{42}$ of $\frac{4\frac{11}{16}}{6\frac{12}{16}}$ to a common denominator.

Ans. $\frac{90}{324}$, $\frac{105}{324}$, and $\frac{125}{324}$.

PROBLEM VIII.

To reduce a series of vulgar fractions to a series of the smallest possible whole numbers, which shall have to each other the same ratio as the given fractions.

Note.—This problem may be applied to the clearing of an algebraic equation from fractions.

RULE.

Find the least common multiple of all the denominators of the given fractions, and multiply each of the fractions by that number.

Ex.—To reduce the vulgar fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$, to the least possible whole numbers, having to each other the same ratio as the given fractions.

$$4\,)\;\cancel{4}\quad\cancel{6}\quad 8\quad 12$$
$$\qquad\qquad\quad 2\quad 3$$
$$4\times2\times3=24 \text{ L. C. M.}$$
$$\frac{3}{\cancel{4}}\times\frac{\overset{6}{\cancel{24}}}{1}=18$$

Here we find the least common multiple of the denominators 4, 6, 8, and 12, to be 24; and multiplying each of the fractions by this number 24, we get the whole numbers 18, 20, 21, and 22, which are the least that

can be obtained, and which also have to each other the same ratio as the given fractions: for $\frac{3}{4} : \frac{5}{6}$ :: 18 : 20; since $\frac{3}{4}\times\frac{20}{1}=\frac{60}{4}=15$, and $\frac{5}{6}\times\frac{18}{1}=\frac{90}{6}=15$; and so ot all the other fractions in the given series.

$$\frac{5}{\cancel{6}} \times \frac{\overset{4}{\cancel{24}}}{1} = 20$$

$$\frac{7}{\cancel{8}} \times \frac{\overset{3}{\cancel{24}}}{1} = 21$$

$$\frac{11}{\cancel{12}} \times \frac{\overset{2}{\cancel{24}}}{1} = 22$$

EXAMPLES FOR PRACTICE.

Reduce the following fractions to a series of whole numbers, the smallest possible, that shall have to each other the same ratio as the fractions.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$. *Ans.* 12, 8, 6, and 3.

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$. *Ans.* 18, 20, 21, and 22.

3. Reduce $\frac{7}{18}$, $\frac{11}{36}$, $\frac{2}{9}$, and $\frac{15}{72}$. *Ans.* 28, 22, 16, and 15.

4. Reduce $\frac{5}{7}$ of $\frac{21}{35}$, $\frac{15}{49}$, and $\frac{9}{24\frac{1}{2}}$. *Ans.* 21, 15, and 18.

PROBLEM IX.

To reduce a vulgar fraction, having an integer of one denomination, into an equivalent fraction having an integer of any other denomination required.

1. As the original value of the given fraction must be invariably the same, whatever may be the integer of which it is to be a part, and as equivalent fractions vary inversely as their integers, it is evident that the given fraction must be made as many times greater as the required integer is less, or as many times less as the required integer is greater than its own.

2. And as fractions vary directly as their numerators, and inversely as their denominators, it is equally evident that a fraction is made greater by multiplying its numerator, and less by multiplying its denominator.

Thus to reduce $\frac{1}{40}$ of a £. to an equivalent fraction of a shilling; as the integer of the required fraction, 1 shilling, is 20 times less than the integer of the given fraction, it is evident that the fraction $\frac{1}{40}$ must be made 20 times greater; we therefore multiply the numerator of the fraction $\frac{1}{40}$ by 20, and get £ $\frac{1 \times 20}{40} = \frac{20}{40} = \frac{1}{2}$, the equivalent fraction of a shilling.

Also, to reduce $\frac{1}{2}$ of a shilling to an equivalent fraction of a £.;—as the integer of the required fraction, £1, is 20 times as great as the integer of the given fraction, it is evident that the fraction $\frac{1}{2}$ must be made 20 times less; we therefore multiply the denominator of the given fraction $\frac{1}{2}$ by 20, and get $s. \frac{1}{2 \times 20} = \frac{1}{40}$, the equivalent fraction of a £.

The required integer may not always be an exact number of times greater or less than the integer of the given fraction, as in the reduction of a fraction of a £. to an equivalent fraction of a guinea.

In this case it is usual to take an intermediate integer between the two, and of which an exact number of units is contained in each of the other integers; and first to reduce the given fraction to a fraction of this intermediate integer, and afterwards to reduce this fraction to the equivalent fraction of the required integer.

Thus, to reduce $\frac{1}{40}$ of a £. to the fraction of a guinea, we take the integer 1 shilling, of which 20 make a £. and 21 a guinea; and reducing the given fraction, $\frac{1}{40}$ of a £., to the fraction of a shilling, £ $\frac{1 \times \cancel{20}}{\underset{2}{\cancel{40}}} = \frac{1}{2}$ of a shilling, we then reduce this fraction, $\frac{1}{2}s.$, to the fraction of a guinea, $s. \frac{1}{2 \times 21} = \frac{1}{42}$ of a guinea.

But without having recourse to this circuitous method

of reduction, we may derive a rule from the nature of fractional quantities, for the direct reduction of the given fraction from its own integer into an equivalent fraction of any other integer whatever.

For although, in whole numbers, it would be absurd to ask how many guineas are contained in a £.; yet there is always a fractional quantity of any integer, however great, contained in any other integer, however small.

Thus, as a £. is equal to 20 shillings, and each of these shillings is the one-and-twentieth part of a guinea, it is evident that $\frac{20}{21}$ of a guinea are contained in a £.; consequently £$\frac{1}{\underset{2}{\cancel{40}}} \times \frac{\cancel{20}}{21} = \frac{1}{42}$ of a guinea.

Also, as a guinea is equal to 21 shillings, and each of these shillings is the twentieth part of a £., it is evident $\frac{21}{20}$ of a £. are contained in a guinea; consequently g. $\frac{1}{\underset{2}{\cancel{42}}} \times \frac{\cancel{21}}{20} = \frac{1}{40}$ of a £.

Here it is obvious that by multiplying the given fraction by such a fractional quantity of the required integer as is contained in its own integer, we make the given fraction either as many times greater as the required integer is less, or as many times less as the required integer is greater than its own; and in either case obtain the equivalent fraction required. Hence the following

GENERAL RULE.

> Multiply the given fraction by such a fractional quantity of the required integer as is contained in its own integer, and the product will be the equivalent fraction required.

Ex. 1.—To reduce $\frac{5}{9}$ of a moidore to an equivalent fraction of a guinea.

Here a moidore is equal to 27 shillings, and a shilling is the one-and-twentieth part of a guinea; consequently,

multiplying the given fraction $\frac{5}{9}$ by $\frac{27}{21}$, we have $\frac{5}{\cancel{9}} \times \frac{\overset{3}{\cancel{27}}}{21} = \frac{15}{21}$, the equivalent fraction of a guinea.

Ex. 2.—To reduce $\frac{15}{21}$ of a guinea to an equivalent fraction of a moidore.

Here a guinea is equal to 21 shillings, and a shilling is the twenty-seventh part of a moidore; consequently, multiplying the given fraction $\frac{15}{21}$ by $\frac{21}{27}$, we have $\frac{\overset{5}{\cancel{15}}}{\cancel{21}} \times \frac{\cancel{21}}{\underset{9}{\cancel{27}}} = \frac{5}{9}$, the equivalent fraction of a moidore.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{3}{5}$ of a £. to an equivalent fraction of a crown.
Ans. $\frac{12}{5}$ crown.

2. Reduce $\frac{2}{3}$ of $\frac{5}{28}$ guinea to the fraction of a shilling.
Ans. $\frac{5}{2}$ shilling.

3. Reduce $\frac{9}{16}$ of a penny to the fraction of a dollar.
Ans. $\frac{1}{96}$ dollar.

4. Reduce 7*s*. 6*d*. to an equivalent fraction of 6*s*. 8*d*.
Ans. $\frac{90}{80} = \frac{9}{8}$.

5. Reduce $\frac{3}{4}$ of 6*s*. 8*d*. to an equivalent fraction of 7*s*. 6*d*.
Ans. $\frac{2}{3}$.

6. Reduce $\frac{3}{5}$ of 17*s*. 6*d*. to an equivalent fraction of 13*s*. 4*d*.
Ans. $\frac{63}{80}$.

7. Reduce $\frac{7}{3120}$ of a lb. troy to the fraction of a dwt.
Ans. $\frac{7}{13}$ dwt.

8. Reduce $\frac{3}{4}$ of an ell Flem. to the fraction of an ell Eng.
Ans. $\frac{9}{20}$ ell Eng.

9. Reduce $\frac{16}{54}$ of a Fr. ell to the fraction of a yard.

Ans. $\frac{4}{9}$ yard.

10. Reduce 6 fur. 24 po. to the fraction of a mile.

Ans. $\frac{33}{40}$ mile.

11. Reduce $\frac{7}{16}$ of a moidore to the fraction of a guinea.

Ans. $\frac{27}{48}$ moidore.

12. Reduce $\frac{8}{9}$ of a tierce to the fraction of a hhd.

Ans. $\frac{16}{27}$ hhd.

13. Reduce $\frac{8}{27}$ of a pipe to the fraction of a tierce.

Ans. $\frac{8}{9}$ tierce.

14. Reduce $\frac{17}{24}$ of a statute mile to the fraction of a geographical mile. *Ans.* $\frac{85}{139}$ geo. mile.

15. Reduce $\frac{45}{80}$ of a crown to the fraction of a dollar.

Ans. $\frac{5}{8}$ dollar.

16. Reduce $\frac{4}{25}$ of an ell Eng. to the fraction of an inch.

Ans. $\frac{36}{5}$ inch.

17. Reduce $\frac{63}{81}$ of an inch to the fraction of a Flem. ell.

Ans. $\frac{7}{243}$ Flem. ell.

18. Reduce $\frac{2}{3}$ of a geographical mile to the fraction of a statute mile.

Ans. $\frac{139}{180}$.

19. Reduce $\frac{3}{5}$ of 3*s*. $9\frac{3}{5}d$. to the fraction of 11*s*. $5\frac{1}{4}d$.

Ans. $\frac{1}{5}$.

PROBLEM X.

To find the integral value of a fraction in the several inferior denominations of its integer.

A fraction may be so small as to have no integral value, even in the lowest denomination into which its integer can be reduced.

Thus, $\frac{1}{1440}$ of a £.; for $\frac{1\times20}{1440}=\frac{1}{72}$ of a shilling, $\frac{1\times\cancel{12}}{\cancel{72}_{6}}=\frac{1}{6}$ of a penny, and $\frac{1\times\cancel{4}^{2}}{\cancel{6}_{3}}=\frac{2}{3}$ of a farthing; and as there is no denomination of the integer £1. less than a farthing, the fraction $\frac{1}{1440}$ of a £. has no integral value.

If the given fraction have an integral value, the fraction of the inferior denomination into which it is reduced will be always an improper fraction, and this improper fraction will always be equal to a whole or mixed number.

Consequently, to find the integral value, we have only to reduce the given fraction to an equivalent fraction of the next inferior denomination, by Prob. 9, and then to reduce this fraction to an equivalent whole or mixed number, by Prob. 4.

If the improper fraction be equal to a whole number, the integral value of the given fraction is fully determined; but if it be equal to a mixed number, there will still remain the fractional part of the mixed number, of which the integral value must be found in the next lower denomination of the integer, as before.

Thus, to find the integral value of the fraction $\frac{4}{5}$ of a £.; reducing this fraction to the fraction of a shilling, we have $\frac{4\times\cancel{20}^{4}}{\cancel{5}}=\frac{16}{1}=16$ shillings, the entire integral value required.

But to find the integral value of the fraction $\frac{5}{8}$ of a £., $\frac{5\times\cancel{20}^{5}}{\cancel{8}_{2}}=\frac{25}{2}=12\frac{1}{2}$ shillings, a mixed number, of which the fractional part $\frac{1}{2}$ must be reduced to an equivalent fraction of the next lower denomination, a penny; thus, $\frac{1\times12}{2}=\frac{12}{2}=6$ pence.

Hence, 12 shillings, the integral value of the given fraction £.$\frac{5}{8}$ in the denomination shillings, and 6 pence, its integral value in the denomination pence, taken together give its full integral value, 12*s.* 6*d.* Hence the

RULE.

1. Reduce the given fraction to an equivalent fraction of the next lower denomination of the integer; and reduce this fraction to an equivalent whole or mixed number.
2. If the result be a whole number, the integral value is found; but if it be a mixed number, reduce the fractional part of the mixed number to a fraction of the next lower denomination, and find its integral value as before.
3. Continue this process till you have reduced into the lowest denomination of the integer, and the several integral values in the several denominations will be the integral value required.

Ex.—To find the integral value of the fraction $\frac{5}{7}$ of a £.

Here, reducing £.$\frac{5}{7}$ to the fraction of the next lower denomination, a shilling, we get the improper fraction, $\frac{100}{7}$; and reducing this improper fraction to an equivalent whole or mixed number, we get 14

$$\frac{5 \times 20}{7} = \frac{100}{7} = 14\tfrac{2}{7} \text{ shillings.}$$

$$\frac{2 \times 12}{7} = \frac{24}{7} = 3\tfrac{3}{7} \text{ pence.}$$

$$\frac{3 \times 4}{7} = \frac{12}{7} = 1\tfrac{5}{7} \text{ farthings.}$$

Hence,

$$£.\frac{5}{7} = 12s.\ 6\tfrac{1}{4}d.\ \frac{5}{7}.$$

shillings and $\frac{2}{7}$ of a shilling; and now reducing the fractional part, $\frac{2}{7}$, of the mixed number, $14\frac{2}{7}$, to the fraction of the next lower denomination, a penny, we get the improper fraction, $\frac{24}{7}$ = 3 pence, and $\frac{3}{7}$ of a penny; and lastly, reducing the fractional part $\frac{3}{7}$, of the mixed number,

$3\frac{3}{7}$, to the fraction of the next lower denomination, a farthing, we get the improper fraction, $\frac{12}{7} = 1$ farthing, and $\frac{5}{7}$ of a farthing; and taking the integral values, 12 shillings, 3 pence, and 1 farthing, we have the whole integral value, 12*s.* 6¼*d.*, to which the remaining fraction, $\frac{5}{7}$, is annexed, as there is no lower denomination of the integer into which it can be reduced.

For merely practical use, we may reduce the numerator of the given fraction into the next lower denomination of the integer, and divide this number by the denominator of the given fraction, reducing every succeeding remainder into the next lower denomination, and dividing by the denominator as before, till we have reduced into the lowest; and the several quotients taken together will be the integral value required.

Thus, to find the integral value of the fraction, $\frac{5}{7}$ of a £.

Here, reducing the numerator 5 into shillings, and dividing by the denominator, 7, we get the quotient, 14 shillings; and reducing the remainder, 2 shillings, into pence, and dividing by the denominator, 7, as before, we get the quotient, 3 pence; and lastly, reducing the remainder, 3 pence, into farthings, and dividing by the denominator, 7, we get the quotient, 1 farthing; and these several quotients, 14 shillings, 3 pence, and 1 farthing, are together the integral value, 12*s.* 3¼*d.* But it is evident that this process is nothing more than Compound Division of whole numbers.

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       £5
       20
7 )   100 ( 14 shillings.
        7
       30
       28
        2
       12
7 )    24 ( 3 pence.
       21
        3
        4
7 )    12 ( 1 farthing.
        7
        5
        -
        7
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EXAMPLES FOR PRACTICE.

1. Find the integral values of the fractions $\frac{3}{5}$ of a £. and $\frac{5}{7}$ of a guinea. *Ans.* 12*s.* and 15*s.*

2. Find the integral values of $\frac{7}{18}$ of a moidore, and $\frac{11}{16}$ of a crown. *Ans.* 10*s.* 6*d.*, and 3*s.* $5\frac{1}{4}$*d.*

3. Find the integral values of $\frac{6}{7}$ of $\frac{9}{16}$ of a cwt., and $\frac{5}{12}$ of $\frac{4}{15}$ of a lb. *Ans.* 1 qr. 26 lb., and 1 oz. $12\frac{4}{9}$ dr.

4. Find the integral values of $\frac{11}{21}$ of a hhd., and $\frac{3}{4}$ of $\frac{8}{9}$ of a tierce. *Ans.* 33 gal., and 28 gal.

5. Find the integral values of $\frac{7}{15}$ of $1\frac{1}{3}$ lb. troy, and $\frac{4}{17}$ of an oz. *Ans.* 7 oz. 9 dwts. 8 grs., and 4 dwt. $16\frac{16}{17}$ grs.

6. Find the integral values of $\frac{13}{24}$ of a week, and $\frac{13}{15}$ of a day. *Ans.* 3 da. 23 ho. 4 min., and 20 ho. 48 min.

ADDITION.

1. Addition of vulgar fractions differs in no respect from addition of whole numbers; for in neither can quantities which are not of the same kind be added into one sum; consequently, the fractional quantities to be added must all be made similar.

2. Hence mixed numbers must be reduced to improper fractions; complex and compound fractions to simple fractions; fractions having different integers, to fractions having one all the same integer; and lastly, fractions having different denominators, to fractions having one common denominator.

3. When all these requisite reductions have been made, the fractional quantities are duly prepared for addition, and their sum is obtained by adding together their several numerators, just like so many quantities in simple addition of whole numbers.

Thus, to add the fractional quantities, $3\frac{3}{4}$ of a £., $\frac{\frac{5}{7}}{4}$ of a guinea; and $\frac{3}{4}$ of $\frac{5}{6}$ of a moidore.

First, we reduce the given quantities into simple frac-

tions; thus, $3\frac{3}{4} = \frac{3\frac{3}{4} \times 4}{1 \times 4} = \frac{15}{4}$ £.; $\frac{\frac{5}{7}}{4} = \frac{\frac{5}{7} \times 7}{7 \times 7} = \frac{5}{28}$ g.; and $\frac{3}{4}$ of $\frac{5}{6} = \frac{\cancel{3} \times 5}{4 \times \underset{2}{\cancel{6}}} = \frac{5}{8}$ m. Thus we have now the simple fractions, $\frac{15}{4}$ £., $\frac{5}{28}$ g., and $\frac{5}{8}$ moidore.

Secondly. As these fractions have different integers, we reduce them to fractions having all the same integer, £1; and as the fraction $\frac{15}{4}$ £. has already the required integer, we have only to reduce the two other fractions: thus,

$\frac{5}{28}$ g. $= \frac{\cancel{5}}{\underset{4}{\cancel{28}}} \times \frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{20}}} = \frac{3}{16}$ £., and $\frac{5}{8}$ m. $= \frac{\cancel{5}}{8} \times \frac{27}{\underset{4}{\cancel{20}}} = \frac{27}{32}$ £.

We have now the simple fractions, $\frac{15}{4}$ £., $\frac{3}{16}$ £., and $\frac{27}{32}$ £., all having the same integer, £1; but as these fractions have different denominators, we lastly reduce them to fractions having one common denominator; and as the denominators 4 and 16 have each a multiple in 32, 32 is their least common multiple, and consequently the common denominator of all the fractions.

Hence $\frac{15}{4} = \frac{15 \times 8}{4 \times 8} = \frac{120}{32}$; $\frac{3}{16} = \frac{3 \times 2}{16 \times 2} = \frac{6}{32}$; and $\frac{27}{32}$, having already the denominator required, needs no reduction.

We have now the fractions $\frac{120}{32}$ £, $\frac{6}{32}$ £., and $\frac{27}{32}$ £.; all simple fractions, all having the same integer £1, and all having one common denominator, 32. They are all, therefore, duly prepared for addition.

We have now only to find their sum by adding together their several numerators; thus, $\frac{120}{32} + \frac{6}{32} + \frac{27}{32} = \frac{120+6+27}{32} = \frac{153}{32}$ £., the sum of the given fractions of which the integral value may be found by Prob. 10.

Hence the

RULE.

1. Prepare the given fractions for addition, as directed above (see Art. 2), and the sum of the numerators, written over the common denominator, will be the sum of the given fractions.

Note.—If there be whole numbers among the quantities to be added, or if the integral parts of the mixed numbers be large, they may be rejected during the operation, and subsequently added to the sum of the fractions.

Ex.—To add 9, $5\frac{3}{4}$, and $11\frac{4}{5}$ we may reject the whole number 9, and the integral parts of the mixed numbers 5 and 11; and adding only the fractions $\frac{3}{4}$ and $\frac{4}{5} = \frac{15}{20} + \frac{16}{20} = \frac{15 + 16}{20} = \frac{31}{20} = 1\frac{11}{20}$, we add to this sum the rejected numbers, 9, 5, and 11, and get $1\frac{11}{20} + 9 + 5 + 11 = 26\frac{11}{20}$, the entire sum.

EXAMPLES FOR PRACTICE.

Add together the following fractions:—

(1.) $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$, and $\frac{3}{4} + \frac{5}{8} + \frac{7}{12}$.
Ans. $2\frac{1}{4}$ and $1\frac{23}{24}$.

(2.) $\frac{7}{8} + \frac{13}{16} + \frac{25}{32} + \frac{45}{64}$, and $\frac{3}{4}$ of $\frac{5}{6}$ of $7\frac{1}{5} + \frac{2}{3}$ of $12\frac{1}{2}$.
Ans. $3\frac{11}{64}$ and $12\frac{5}{6}$.

(3.) $7\frac{3}{5} + 5\frac{7}{10} + \frac{7}{15}$, and $\frac{5}{6} + 7\frac{1}{2} + \frac{2}{3}$ of $\frac{3}{4}$ of $19\frac{5}{6}$.
Ans. $13\frac{23}{30}$ and $18\frac{1}{4}$.

(4.) $\frac{12\frac{1}{2}}{17} + \frac{14}{22\frac{2}{3}} + \frac{12\frac{1}{5}}{13\frac{6}{10}}$, and $\frac{3}{5}$ of $\frac{15}{54} + \frac{7}{16}$ of $12\frac{4}{5}$.
Ans. $2\frac{1}{4}$ and $5\frac{23}{30}$.

(5.) $\frac{3}{4}$ of a shilling $+ \frac{7}{8}$ crown $+ \frac{5}{14}$ of a guinea.
Ans. 12*s.* $7\frac{1}{2}$*d.*

(6.) $\frac{2}{7}$ of £15 + $3\frac{3}{7}$ £. + $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ £. + $\frac{2}{3}$ of $\frac{3}{7}$ shillings.
Ans. £7 17*s.* 5*d.*$\frac{1}{7}$.

(7.) $\frac{17}{27}$ of a dollar + $\frac{119}{189}$ guinea + $\frac{17}{54}$ moidore.

Ans. £1 „ 4*s.* 6½*d.* $\frac{2}{3}$.

(8.) $\frac{3}{16}$ lb. + $\frac{45}{64}$ oz. + $\frac{45}{48}$ dwts., and $\frac{7}{16}$ oz. + $3\frac{1}{3}$ dwts. + $17\frac{3}{4}$ grs. *Ans.* 3 oz., and 12 dwt. 14 gr. $\frac{1}{2}$.

(9.) $\frac{2}{3}$ Flem. ell + $\frac{3}{4}$ yd. + $\frac{4}{5}$ Eng. ell + $\frac{5}{6}$ Fr. ell, and $\frac{7}{8}$ in. + $\frac{23}{24}$ na. + $\frac{15}{16}$ qrs. *Ans.* $3\frac{1}{3}$ yds., and 1 qr. 1 na. $\frac{7}{72}$.

(10). $\frac{3}{64}$ ton + $\frac{7}{16}$ cwt. + $\frac{5}{8}$ qr., and $\frac{17}{32}$ lb. + $\frac{37}{42}$ oz. + $\frac{48}{63}$ dr. *Ans.* 1 cwt. 2 qr. 3 lb. 8 oz., and 9 oz. $6\frac{6}{7}$ dr.

(11.) $\frac{11}{12}$ mile + $\frac{7}{15}$ fur. + $\frac{8}{9}$ po., and $\frac{28}{33}$ po. + $\frac{38}{54}$ yd.

Ans. 7 fur. 32 po. 4 yds. 2 ft. 8 in., and 5 yds. $1\frac{1}{3}$ ft.

(12.) $\frac{5}{7}$ tun + $\frac{8}{21}$ pipe + $\frac{11}{14}$ hhd., and $\frac{3}{7}$ tier. + $\frac{5}{8}$ gal. + $\frac{9}{10}$ pint.

Ans. 1 tun $25\frac{1}{2}$ gal., and 18 gal. 2 qt. $1\frac{2}{10}$ pts.

(13.) $\frac{25}{219}$ year + $\frac{16}{27}$ day + $\frac{35}{54}$ ho., and $\frac{7}{15}$ we. + $4\frac{2}{3}$ day + $\frac{24}{45}$ ho.

Ans. 42 da. 6 ho. 52 min. $13\frac{1}{3}$ sec., and 7 da. 21 ho. 20 min.

(14.) $\frac{7}{24}$ of $\frac{12}{56}$ day + $\frac{1}{15}$ of $17\frac{1}{7}$ ho. + $\frac{25}{36}$ of $50\frac{2}{3}$ min.

Ans. 3 ho. 3 min. $34\frac{2}{7}$ sec.

SUBTRACTION.

1. Subtraction of Vulgar Fractions differs in no respect from subtraction of whole numbers. Having therefore prepared the fractional quantities as in Addition (see Add., Art. 2), we have only to subtract the numerator of the smaller fraction from the numerator of the greater, and the remainder written over the common denominator will be the difference of the given fractions.

Thus, to subtract $\frac{3}{4}$ of $\frac{5}{6}$ of a guinea from $\frac{4}{5}$ of $\frac{5}{7}$ of a moidore.

First. Reducing the compound fractions to simple fractions, we have $\frac{3}{4}$ of $\frac{5}{6} = \frac{\cancel{3}\times 5}{4\times \cancel{6}_{2}} = \frac{5}{8}$; and $\frac{4}{5}$ of $\frac{5}{7} = \frac{4\times \cancel{5}}{\cancel{5}\times 7} = \frac{4}{7}$; hence we have the simple fractions of $\frac{5}{8}$ g. and $\frac{4}{7}$ m.

Secondly. Reducing these fractions which have different integers to fractions having both the same integer, 1 shilling, we have $\frac{5}{8}$ g. $= \frac{5}{8} \times \frac{21}{1} = \frac{105}{8}$ *s.* and $\frac{4}{7}$ m. $= \frac{4}{7} \times \frac{27}{1} = \frac{108}{7}$ *s.*; hence we have the fractions $\frac{105}{8}$ *s.* and $\frac{108}{7}$ *s.* both having the same integer, 1 shilling.

Thirdly. Reducing these fractions which have different denominators to fractions having one common denominator, we have $\frac{105}{8} = \frac{105\times 7}{8\times 7} = \frac{735}{56}$; and $\frac{108}{7} = \frac{108\times 8}{7\times 8} = \frac{864}{56}$; both having one common denominator, 56.

We have now the fractions $\frac{735}{56}$ *s.* and $\frac{864}{56}$ *s.*, both simple fractions, both having the same integer, 1 shilling, and both having one common denominator, 56.

And now, as the given fractions are duly prepared for subtraction, we have only to subtract the numerator of the smaller, 735, from the numerator of the greater, 864; and the remainder, 129, written over the common denominator, is the difference of the given fractions.

Thus $\frac{864}{56} - \frac{735}{56} = \frac{864-735}{56} = \frac{129}{56}$, the difference of the given fractions, of which the integral value may be found by Problem 10.

No further Rule is requisite.

EXAMPLES FOR PRACTICE.

Find the difference of the following Fractions:—

(1.) $\frac{3}{4} - \frac{2}{3}$, $\frac{5}{7} - \frac{3}{5}$, $\frac{11}{12} - \frac{10}{11}$, and $\frac{4}{5} - \frac{3}{4}$.

Ans. $\frac{1}{12}$, $\frac{4}{35}$, $\frac{1}{132}$, and $\frac{1}{20}$.

(2.) $\frac{3}{5}$ of $\frac{5}{7}$ $-$ $\frac{3}{8}$ of $\frac{8}{15}$, and $\frac{5}{6}$ of $\frac{7}{8}$ $-$ $\frac{3}{4}$ of $\frac{5}{8}$.

Ans. $\frac{8}{35}$ and $\frac{25}{96}$.

(3.) $\frac{2}{3}$ of $17\frac{1}{2}$ $-$ $\frac{1}{9}$ of 19, and $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ $-$ $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

Ans. $9\frac{5}{9}$ and $\frac{6}{35}$.

(4.) $\frac{9}{11}$ of $\frac{7}{27}$ of 22 $-$ $\frac{7}{16}$ of $\frac{8}{21}$ of 17, and $23\frac{1}{4}$ $-$ $17\frac{7}{8}$.

Ans. $1\frac{5}{6}$ and $5\frac{3}{8}$.

(5.) $\frac{7}{8}$ of $\frac{13}{14}$ of $\frac{15\frac{3}{7}}{22\frac{2}{7}}$ $-$ $\frac{9}{13}$ of $\frac{16}{27}$ of $\frac{30\frac{1}{3}}{42\frac{2}{3}}$, and $9\frac{3}{16}$ $-$ $7\frac{5}{24}$.

Ans. $\frac{13}{48}$ and $1\frac{47}{48}$.

(6.) $\frac{7}{12}$ guin. $-$ $\frac{5}{16}$ £., and $\frac{3}{16}$ £. $-$ $\frac{4}{15}$ crown.

Ans. 6*s.* 1*d.* and 2*s.* 5*d.*

(7.) $\frac{19}{54}$ moidore $-$ $\frac{17}{42}$ guin., and $\frac{9}{10}$ of 7*s.* 6*d.* $-$ $\frac{5}{16}$ of 6*s.* 8*d.*

Ans. 1*s.* and 4*s.* 8*d.*

(8.) $\frac{3}{16}$ of $\frac{7}{9}$ lb. troy $-$ $\frac{5}{12}$ of $\frac{9}{10}$ oz., and $\frac{13}{96}$ dwt. $-$ $\frac{15}{16}$ gr.

Ans. 1 oz. 7 dwt. 12 gr., and $2\frac{5}{16}$ gr.

(9.) $\frac{12}{27}$ Hhds. $-$ $\frac{8}{15}$ tierce, and $\frac{5}{6}$ pipe $-$ 75 gal.

Ans. 5 gal. $2\frac{3}{5}$ qt., and 30 gall.

(10.) $\frac{3}{14}$ of $\frac{7}{10}$ ton $-$ $\frac{5}{17}$ of $\frac{34}{35}$ cwt., and $\frac{6}{7}$ of $\frac{2}{9}$ qur. $-$ $\frac{7}{16}$ of $4\frac{3}{4}$ lb.

Ans. 2 cwt. 2 qrs. 24 lb., and 3 lb. 4 oz. $1\frac{1}{3}$ drs.

(11.) $\frac{14}{15}$ of $\frac{8}{21}$ league $-$ $\frac{3}{8}$ of $\frac{3\frac{1}{2}}{4\frac{1}{2}}$ mile, and $\frac{17}{24}$ mi. $-$ $\frac{24}{75}$ of $28\frac{1}{8}$ po. *Ans.* 6 fur. 8 po., and 5 fur. 17 po. 3 yds. 2 ft.

(12.) $\frac{17}{219}$ year $-$ $\frac{37}{42}$ of 28 day, and $\frac{7}{24}$ mo. $-$ $\frac{8}{15}$ wk.

Ans. 3 days. 16 hrs., and 5 days 0 hrs. 24 min.

MULTIPLICATION.

1. As fractions vary directly as their numerators, and inversely as their denominators, it is evident that if the numerator only be multiplied, the fraction will be made greater; or if the denominator only be multiplied, the fraction will be made less.

2. It is also evident, from the nature of multiplication, that the product must be either as many times greater or as many times less than the multiplicand, as the multiplier is greater or less than a unit.

Hence, if the numerator and denominator of the multiplicand be multiplied respectively by the numerator and denominator of the multiplier, the product will be either as many times greater, or as many times less than the multiplicand, as the multiplier is greater or less than a unit.

Thus, to multiply $\frac{7}{8}$ by $\frac{2}{4}$; $\frac{7}{8} \times \frac{2}{4} = \frac{7\times\cancel{2}}{\underset{4}{\cancel{8}}\times 4} = \frac{7}{16}$, which product is twice as small as the multiplicand $\frac{7}{8}$; for in obtaining it we made the fraction $\frac{7}{8}$ twice as great by multiplying its numerator by 2, and also four times as small by multiplying its denominator by 4.

Consequently, as the decrease is just twice as great as the increase, it is evident that the product $\frac{7}{16}$ is just twice as small as the multiplicand $\frac{7}{8} = \frac{14}{16}$; or just as many times less as the multiplier $\frac{2}{4}$ is less than a unit.

Also, to multiply $\frac{7}{8}$ by $\frac{4}{2}$; $\frac{7}{8} \times \frac{4}{2} = \frac{7\times\cancel{4}}{\underset{2}{\cancel{8}}\times 2} = \frac{7}{4}$, which product is twice as great as the multiplicand; for in obtaining it, we made the fraction $\frac{7}{8}$ four times as great by multiplying its numerator by 4, and also twice as small by multiplying its denominator by 2.

Consequently, as the increase is just twice as great as the decrease, it is evident that the product $\frac{7}{4} = \frac{14}{8}$,

is just twice as great as the multiplicand $\frac{7}{8}$, or just as many times greater as the multiplier $\frac{4}{2}$ is greater than a unit.

Thus it is evident that in the first example $\frac{7}{8} \times \frac{2}{4} = \frac{7 \times \cancel{2}}{\underset{2}{\cancel{8}} \times 4} = \frac{7}{16}$, in which the multiplier is twice as small as a unit, the product $\frac{7}{16}$ is just twice as small as the multiplicand $\frac{7}{8} = \frac{14}{16}$.

And also in the second example $\frac{7}{8} \times \frac{4}{2} = \frac{7 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 2} = \frac{7}{4}$, in which the multiplier is twice as great as a unit, the product $\frac{7}{4} = \frac{14}{8}$ is just twice as great as the multiplicand $\frac{7}{8}$.

Consequently, in both, the product is just as many times greater or as many times less than the multiplicand, as the multiplier is greater or less than a unit, and is therefore the true product.

Hence the

RULE.

Reduce mixed numbers and complex fractions to simple fractions, and the product of the numerators written over the product of the denominators, will be the product of the given fractions.

Ex.—To multiply $3\frac{1}{2}$, $\frac{4\frac{2}{3}}{7}$, and $\frac{3}{4}$ of $\frac{4}{5}$.

Here $3\frac{1}{2} = \frac{3\frac{1}{2} \times 2}{1 \times 2} = \frac{7}{2}$, $\frac{4\frac{2}{3}}{7} = \frac{4\frac{2}{3} \times 3}{7 \times 3} = \frac{14}{21}$, and $\frac{3}{4}$ of $\frac{4}{5}$ is sufficiently prepared for multiplication; hence $\frac{7}{2} \times \frac{14}{21} \times \frac{3}{4}$ of $\frac{4}{5} = \frac{\overset{7}{\cancel{7}} \times \cancel{14} \times \cancel{3} \times \cancel{4}}{\cancel{2} \times \underset{3}{\cancel{21}} \times \cancel{4} \times 5} = \frac{7}{5}$, the product of the given fractions.

EXAMPLES FOR PRACTICE.

Find the products of the following fractions :—

(1.) $\frac{3}{4}\times\frac{7}{8}$, $\frac{5}{7}\times\frac{11}{15}$, $\frac{84}{90}\times\frac{18}{42}$, and $\frac{17}{35}\times\frac{15}{68}$.

Ans. $\frac{21}{32}$, $\frac{11}{21}$, $\frac{2}{5}$, and $\frac{3}{28}$.

(2.) $\frac{5}{6}$ of $\frac{7}{15}\times\frac{7}{9}$ of $\frac{15}{28}$, and $\frac{13}{27}$ of $\frac{9}{26}\times\frac{15}{16}$ of $\frac{32}{75}$.

Ans. $\frac{35}{216}$ and $\frac{1}{15}$.

(3.) $\frac{5}{7}$ of $\frac{3}{4}$ of $3\frac{1}{2}\times\frac{3}{5}$ of $\frac{4}{9}$ of $7\frac{1}{5}$, and $17\frac{1}{2}\times18\frac{1}{4}$.

Ans. $3\frac{3}{5}$ and $319\frac{3}{8}$.

(4.) $\frac{16\frac{4}{5}}{19\frac{1}{5}}\times\frac{14}{15\frac{3}{4}}$, and $\frac{15}{28}$ of $\frac{7}{30}\times\frac{19}{128}$ of $\frac{32}{57}$ of $57\frac{3}{5}$.

Ans. $\frac{7}{9}$ and $\frac{3}{5}$.

(5.) $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{7}{8}$ of $100\times\frac{2}{3}$ of $\frac{9}{10}$ of $\frac{28}{50}$, and $\frac{7\frac{1}{2}}{12}\times\frac{12}{13\frac{1}{3}}$.

Ans. $11\frac{13}{20}$ and $\frac{9}{16}$.

(6.) $\frac{27\frac{3}{8}}{54}\times\frac{18}{24\frac{1}{3}}$, and $\frac{2}{11}$ of $\frac{22}{35}$ of $17\frac{1}{2}\times\frac{3}{7}$ of $\frac{2\frac{1}{3}}{11}$.

Ans. $\frac{3}{8}$ and $\frac{2}{11}$.

(7.) $\frac{4}{5}$ of $\frac{10}{12}$ of $\frac{24}{25}$ of $5\times\frac{9}{10}$ of $\frac{10}{11}$ of $\frac{11}{12}$ of 9.

Ans. $21\frac{3}{5}$.

(8.) $\frac{7}{19}$ of $\frac{34}{49}$ of $\frac{12}{17}$ of $4\frac{27}{36}$, $\times\frac{9}{17}$ of $\frac{26}{27}$ of $\frac{11}{13}$ of $5\frac{2}{3}$.

Ans. $2\frac{8}{21}$.

DIVISION.

1. As Division is just the reverse of Multiplication, it will be evident from what has been said on Multiplication of Vulgar Fractions, that if the numerator only be divided, the fraction will be made less; or if the denominator only be divided the fraction will be made greater.

2. It is also evident, from the nature of Division, that the quotient must be either as many times greater than the dividend as the divisor is less, or as many times less than the dividend as the divisor is greater than a single unit.

3. Hence, if the numerator and also the denominator of the dividend be divided by the numerator and denominator of the divisor respectively, the quotient of the numerator written over the quotient of the denominator will be the true quotient of the fraction.

Thus, to divide the fraction $\frac{8}{12}$ by the fraction $\frac{2}{4}$; $\frac{8}{12}\div\frac{2}{4}=\frac{8\div2}{12\div4}=\frac{4}{3}=\frac{16}{12}$; which is just twice as great as the dividend, $\frac{8}{12}$, or just as many times greater as the divisor, $\frac{2}{4}$, is less than a unit, and is consequently the true quotient.

Also, to divide the fraction $\frac{8}{12}$ by the fraction $\frac{4}{2}$; $\frac{8}{12}\div\frac{4}{2}=\frac{8\div4}{12\div2}=\frac{2}{6}=\frac{4}{12}$; which is just twice as small as the dividend, $\frac{8}{12}$, or just as many times less as the divisor, $\frac{4}{2}$, is greater than a unit, and is consequently the true quotient.

4. When the numerator and denominator of the dividend can both be divided by the numerator and denominator of the divisor without any remainder, this is decidedly the best rule for the division of vulgar fractions, as it gives the quotient in lower terms than any other.

5. But if there be any remainder from either of the terms of the dividend, the quotient will be a complex fraction; as $\frac{8}{12}\div\frac{3}{4}=\frac{8\div3}{12\div4}=\frac{2\frac{2}{3}}{3}$; hence a more general rule is requisite, which will in every case give a simple fraction for the quotient.

Such a rule may be easily deduced from the following considerations:—

6. If the divisor be a proper fraction, as $\frac{2}{4}$, which is just twice as small as a unit, it is evident that if this divisor

be inverted we shall have the improper fraction, $\frac{4}{2}$, which is just twice as great as a unit:

Consequently, if we multiply the dividend by this inverted divisor, $\frac{4}{2}$, the product will be just twice as great as the dividend, or just as many times greater as the divisor, $\frac{2}{4}$, is less than a unit.

Thus, to divide the fraction, $\frac{8}{12}$, by the fraction, $\frac{2}{4}$:

$$\frac{8}{12} \div \frac{2}{4} = \frac{8}{12} \times \frac{4}{2} = \frac{\overset{4}{\cancel{8}} \times \cancel{4}}{\underset{3}{\cancel{12}} \times \cancel{2}} = \frac{4}{3} = \frac{16}{12},$$ which is just twice as great as the dividend, $\frac{8}{12}$; or just as many times greater as the divisor, $\frac{2}{4}$, is less than a unit; and is consequently the true quotient.

7. If the divisor be an improper fraction, as $\frac{4}{2}$, which is just twice as great as a unit, it is evident that if we invert this divisor we shall have the proper fraction, $\frac{2}{4}$, which is just twice as small as a unit:

Consequently, if we multiply the dividend by this inverted divisor, $\frac{2}{4}$, the product will be just twice as small as the dividend, or just as many times less as the divisor, $\frac{4}{2}$, is greater than a unit.

Thus, to divide the fraction, $\frac{8}{12}$, by the fraction, $\frac{4}{2}$;

$$\frac{8}{12} \div \frac{4}{2} = \frac{8}{12} \times \frac{2}{4} = \frac{\overset{2}{\cancel{8}} \times \cancel{2}}{\underset{6}{\cancel{12}} \times \cancel{4}} = \frac{2}{6} = \frac{4}{12},$$ which is just twice as small as the dividend, $\frac{8}{12}$; or just as many times less as the divisor, $\frac{4}{2}$, is greater than a unit; and is consequently the true quotient.

Hence, it is evident that if we multiply the dividend by the inverted divisor, the product will, in every case, be either as many times greater than the dividend as the

divisor is less, or just as many times less than the dividend as the divisor is greater than a unit, and will consequently be the true quotient. Hence, the

RULE.

1. Divide the numerator and denominator of the dividend respectively by the numerator and denominator of the divisor; and the quotient of the numerator written over the quotient of the denominator will be the quotient required.
2. Or, if the numerator and denominator of the dividend cannot both be divided without a remainder, multiply the dividend by the inverted divisor, and the product will be the quotient required.

EXAMPLES FOR PRACTICE.

Find the quotients of the following: —

(1.) $\frac{14}{25} \div 7$, $\frac{24}{35} \div \frac{6}{7}$, $\frac{9}{16} \div \frac{3}{4}$, and $\frac{12}{49} \div \frac{3}{7}$.

Ans. $\frac{2}{25}$, $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{4}{7}$.

(2.) $\frac{4}{5} \div \frac{16}{20}$, $\frac{7}{12} \div \frac{2}{5}$, $\frac{35}{36} \div \frac{7}{8}$, and $\frac{76}{171} \div \frac{4}{19}$.

Ans. 1, $1\frac{11}{24}$, $1\frac{1}{9}$, and $2\frac{1}{9}$.

(3.) $\frac{4}{15} \div \frac{16}{20}$, $3\frac{1}{2} \div 2\frac{1}{3}$, and $\frac{4}{5}$ of $7\frac{1}{2} \div \frac{5}{6}$ of $\frac{3}{4}$.

Ans. $\frac{1}{3}$, $1\frac{1}{2}$, and $9\frac{3}{5}$.

(4.) $50 \div \frac{5}{9}$ of $\frac{3}{35}$ of $\frac{1}{21}$ of $12\frac{3}{5}$, and $17\frac{3}{5} \div 4\frac{2}{5}$.

Ans. 1750 and 4.

(5.) $\frac{2\frac{2}{5}}{3\frac{1}{5}} \div \frac{7}{16}$ of $\frac{28}{35}$, and $\frac{5}{7}$ of $9\frac{4}{5} \div \frac{3}{4}$ of $\frac{3}{7}$ of $\frac{4}{9}$.

Ans. $2\frac{1}{7}$ and 49.

(6.) $\frac{23}{24}$ of $\frac{8}{11\frac{1}{2}} \div \frac{5}{21}$ of $\frac{24\frac{1}{2}}{35}$, and $\frac{7}{16}$ of $\frac{112}{96} \div \frac{27}{48}$ of $\frac{7}{108}$.

Ans. 4 and 14.

(7.) $\frac{3}{17} \div \frac{3}{1700}$, $\frac{3}{1700} \div \frac{3}{17}$, $\frac{9}{50} \div \frac{18}{250000}$, and $1 \div \frac{1}{125}$.

Ans. 100; $\frac{1}{100}$, 2500, and 125.

(8.) $\frac{3\frac{5}{6}}{19\frac{1}{6}} \div \frac{2\frac{3}{5}}{18\frac{1}{5}}$; and $\frac{2}{3}$ of $\frac{23\frac{1}{3}}{35\frac{5}{9}} \div \frac{1}{4}$ of $\frac{11\frac{5}{7}}{46\frac{12}{17}}$.

Ans. $1\frac{2}{5}$, and $7\frac{1}{10}$.

PROPORTION.

1. Proportion of Vulgar Fractions differs in no respect from proportion of whole numbers, except that the quantities concerned are fractional.

2. We have, therefore, only to reduce the fractional quantities into their simplest form; to reduce those of which the ratio is given in the question, to fractions of the same integer; and to write down the three given terms of the proportion thus prepared, exactly as in proportion of whole numbers.

3. Having thus "stated the question," we have only to multiply together the second and third terms of the proportion; and the quotient of their product divided by the first term, will be the fourth term of the proportion.

4. As in division of fractions, we have shown that the quotient is more conveniently obtained by multiplying by the inverted divisor, we have only to invert the first term, and the product of the three terms will be the fourth term required.

Ex.—If $\frac{3}{4}$ of $\frac{5}{6}$ of an ounce troy, cost $\frac{5}{7}$ of $\frac{9}{10}$ of a guinea; what will be the value of $\frac{7}{8}$ of $5\frac{5}{14}$ of a pennyweight?

Here, reducing the fractional quantities to their simplest form, we have $\frac{3}{4}$ of $\frac{5}{6} = \frac{\cancel{3}\times 5}{4\times \cancel{6}_{2}} = \frac{5}{8}$ oz.; $\frac{5}{7}$ of $\frac{9}{10} = \frac{\cancel{5}\times 9}{7\times \cancel{10}_{2}}$ $= \frac{9}{14}$ g.; and $\frac{7}{8}$ of $5\frac{5}{14} = \frac{7}{8}$ of $\frac{75}{14} = \frac{\cancel{7}\times 75}{8\times \cancel{14}_{2}} = \frac{75}{16}$ dwt.

Secondly, reducing the fractions $\frac{5}{8}$ oz. and $\frac{75}{16}$ dwt., of

which the ratio is given in the question, to fractions having the same integer, we have $\frac{5}{8}$ oz. $= \frac{5\times 20}{8} = \frac{25}{2}$ dwt.

Lastly, writing down the three terms of the proportion thus prepared, exactly as in proportion of whole numbers, we have the given ratio of weights, $\frac{25}{2} : \frac{75}{16} :: \frac{9}{14}$ g.

And now, inverting the first term and multiplying the three terms together, we have $\frac{\cancel{2}}{\cancel{25}} \times \frac{\overset{3}{\cancel{75}}}{\underset{8}{\cancel{16}}} \times \frac{9}{14} = \frac{27}{112}$ g., the fourth term required, and finding the integral value by Problem 10, we have $\frac{27}{112}$ g. $=$ 5*s*. 0¾*d*.

EXAMPLES FOR PRACTICE.

1. If $\frac{3}{5}$ of a yard cost $\frac{11}{12}$ of a £., what will be the value of $\frac{7}{15}$ of a yard, at the same rate? *Ans*. 14*s*. 3*d*.$\frac{1}{3}$

2. If $\frac{3}{16}$ of an ounce cost $\frac{3}{40}$ of a guinea, what will $\frac{3}{5}$ of 2½ dwts. cost? *Ans*. 6*s*. 3½*d*.$\frac{1}{5}$

3. If $\frac{4}{11}$ of 2½ cwt. cost £2¾, what must be paid for $\frac{2}{3}$ of $\frac{3}{11}$ ton. *Ans*. £11.

4. If $\frac{5}{8}$ of a gallon cost $\frac{5}{8}$£., what will be the value of $\frac{5}{9}$ of a tun of wine? *Ans*. £140.

5. How much in length that is 3⅗ inches broad wil make a square foot? *Ans*. 40 inches.

6. If, when flour is $\frac{11}{40}$£. per peck, the penny loaf weigh $6\frac{8}{10}$ of an ounce, what will it weigh when flour is $\frac{22}{60}$£. per peck? *Ans*. 5 oz. 2⅗ drs.

7. If $\frac{13}{15}$ of an Eng. ell cost $8\frac{2}{3}$ of a shilling, what will be the value of $\frac{2}{5}$ of $3\frac{3}{4}$ of a Fl. ell? *Ans.* 9 sh.

8. If, when the day is $12\frac{4}{5}$ hours long, a house may be built in $17\frac{1}{7}$ days; in what time can it be built when the day is $13\frac{1}{2}$ hours long? *Ans.* $16\frac{16}{63}$ days.

9. How much in breadth that is $17\frac{1}{7}$ inches long will make a square foot? *Ans.* $8\frac{2}{5}$ inches.

10. Bought $3\frac{1}{2}$ pieces of cloth, each piece $24\frac{1}{8}$ Eng. ells long, at $\frac{7}{40}$£. per ell Fl., what is the value of the whole?

Ans. £13 „ 8*s.* $5\frac{1}{2}$*d.*

11. If 24 men will reap a field when the day is $13\frac{1}{2}$ hours long, how many men will do the same when the day is 18 hours long? *Ans.* 18 men.

12. How much in length that is $2\frac{1}{5}$ yards wide will make a square pole? *Ans.* $13\frac{3}{4}$ yards.

13. If $3\frac{3}{4}$ yards of cloth $\frac{3}{4}$ yard wide will make a dress, how many yards $1\frac{1}{4}$ yard wide will be sufficient for the purpose? *Ans.* 2 yds. 1 qr.

14. A person, having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{16}$ of his share for $\frac{3}{4}$ of £288, what is the whole mine worth?

Ans. £1920.

15. If $\frac{3}{7}$ of $\frac{7}{9}$ of $7\frac{1}{2}$ yards cost $\frac{3}{8}$ of $\frac{14}{15}$ of a crown, what will $\frac{2}{3}$ of $\frac{5}{8}$ of $3\frac{1}{5}$ Eng. ells cost? *Ans.* 1*s.* 2*d.*

16. How much in width that is $6\frac{6}{8}$ yards long will make a square pole? *Ans.* $4\frac{13}{27}$ yards.

17. A person completes a journey in $21\frac{1}{2}$ days at the rate of $3\frac{3}{4}$ miles per hour, in what time will he travel $\frac{2}{3}$ of $\frac{3}{5}$ of the same journey at the rate of $5\frac{1}{4}$ miles per hour? *Ans.* $6\frac{1}{7}$ days.

18. Bought $3\frac{3}{4}$ pieces of silk, each $14\frac{3}{5}$ yards long, for $\frac{8}{9}$ of 100 moidores, how many Eng. ells can I buy for $\frac{2}{3}$ of $\frac{5}{7}$ of 100 guineas?

Ans. $18\frac{1}{4}$ Eng. ells.

19. A. can do a piece of work in 10 days and B. in 15 days, in how many days will both do the same, working together? *Ans.* 6 days.

20. Bought $\frac{5}{6}$ of $\frac{9}{10}$ of a pipe of wine for $\frac{3}{5}$ of $\frac{5}{8}$ of £120, what must I pay for $\frac{3}{4}$ of $\frac{8}{9}$ of a tierce?

Ans. £13 „ 6*s.* 8*d.*

21. A. did a piece of work by himself in 10 days, and with the assistance of B. in 6 days; what part of the work was done by each? *Ans.* A. did $\frac{3}{5}$, and B. $\frac{2}{5}$.

22. A person after losing $\frac{3}{17}$ of an estate, sold $\frac{2}{7}$ of the remainder for $\frac{3}{5}$ of £500; what was the value of the whole estate? *Ans.* £1275.

23. A. can do a piece of work in 10 days, B. in 12 days, and C. in 15 days; in what time will they do it all working together? *Ans.* 4 days.

24. A. can do a piece of work in 10 days, and with the assistance of B. in $5\frac{5}{11}$ days; and A. and B. with the assistance of C. can do the same in 4 days; what part of the work was done by each in the four days?

Ans. A. did $\frac{2}{5}$, B. $\frac{1}{3}$, and C. $\frac{4}{15}$.

DECIMAL FRACTIONS.

1. A decimal fraction is a fraction of which the integer is divided into ten equal parts or tenths, which tenths may be subdivided into tenths of tenths or hundredths, and these hundredths in like manner into tenths of hundredths or thousandths, &c. &c. &c.

2. A decimal fraction is expressed by its numerator

only, written on the right hand of the place of units, from which it is separated by a point (·), and the denominator will be always the unit 1 with as many ciphers on the right hand, as there are places in the numerator.

Thus, the decimal fraction three-tenths is expressed by the numerator 3, written in the first place to the right hand of the point; as, ·3, which is the place of tenths, and shows that the denominator consists of the unit 1 and one cipher on the right hand; hence, $\cdot 3 = \frac{3}{10}$.

The decimal fraction four hundredths is expressed by the numerator 4, written in the second place to the right hand of the point; as, ·04, which is the place of hundredths, and shows that the denominator consists of the unit 1 and two ciphers on the right hand; hence, $\cdot 04 = \frac{4}{100}$.

And the decimal fraction five thousandths is expressed by the numerator 5, written in the third place to the right hand of the point; as, ·005, which is the place of thousandths, and shows that the denominator consists of the unit 1 and three ciphers on the right hand; hence, $\cdot 005 = \frac{5}{1000}$.

3. Hence, it is evident that $\cdot 3 = \frac{3}{10}$, $\cdot 04 = \frac{4}{100}$, and $\cdot 005 = \frac{5}{1000}$; in each of which fractions, the denominator consists of the unit 1 with just as many ciphers on the right hand, as there are places in the numerator.

4. Consequently, the denominator of a decimal fraction is not written; for the numerator alone shows not only the number of parts contained in the fraction, but also, by the place in which it is written, the exact magnitude of each of these parts.

5. Ciphers annexed to the right hand of the numerator of a decimal fraction, make no alteration in its magnitude; for they do not alter the situation of the figures with respect to the decimal point or place of units, by their distance from which, their local values are determined.

Thus, the decimals ·5, ·50, ·500, have all the same magnitude; for in all the figure 5 is still in the first place

to the right hand of the decimal point: consequently, $\cdot 5 = \frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$, &c. &c. &c.

6. But the placing of ciphers on the left hand of the numerator of a decimal fraction, by removing the figures farther from the decimal point or place of units towards the right hand, decreases their magnitude ten times for every cipher so placed: thus, $\cdot 5 = \frac{5}{10}$, $\cdot 05 = \frac{5}{100}$, and $\cdot 005 = \frac{5}{1000}$.

7. When there are several figures in the numerator of a decimal fraction, each of them has a different local value, and consequently a different denominator; as in the decimal fraction $\cdot 345$, the 3 in the first place are $\frac{3}{10}$, the 4 in the second place are $\frac{4}{100}$, and the 5 in the third place are $\frac{5}{1000}$.

But these three fractions may be all regarded as one fraction; for $\frac{3}{10} = \frac{300}{1000}$, and $\frac{4}{100} = \frac{40}{1000}$, which have both the same denominator as the fraction $\frac{5}{1000}$; consequently, $\frac{300}{1000} + \frac{40}{1000} + \frac{5}{1000} = \frac{300+40+5}{1000} = \frac{345}{1000} = \cdot 345$, which is one fraction, having the numerator 345, and the denominator 1000, in which there are just as many ciphers as there are places in the numerator.

8. Hence, by taking all the figures in the numerator of a decimal fraction, and reading them as one number according to their several local values, as in numeration of whole numbers, the several parts of the decimal fraction are all made similar to those of the lowest rank.

For the figure 3, when read in combination with the figures 4 and 5, will be 300, and the figure 4, in combination with the figure 5, will be 40; consequently each of them, when taken collectively, will be just as many times greater than when taken singly, as the denominator of the lowest fraction is greater than its own denominator.

Thus by the mere act of enumeration the several fractions are reduced to equivalent fractions, all having one

common denominator, a process which in vulgar fractions is comparatively tedious; and in this facility of reduction to a common denominator, consists the principal advantage of decimal fractions.

9. If we begin at any place above the place of units, and continue the numeration table in a descending series to any place below it, we shall have a series of local values corresponding exactly to the several denominators of decimal fractions.

Thus, 1000, 100, 10, 1, ·1, ·01, ·001, or $\frac{1000}{1}, \frac{100}{1}, \frac{10}{1}, \frac{1}{1}, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$. Here we find that throughout the whole series, both fractional and integral, ten in any lower place are equal to one in the next higher place.

10. Hence it is evident, that all the decimal parts of the integer have to each other the same ratio as the several local values in whole numbers; and consequently that the several rules of addition, subtraction, multiplication, and division of decimal fractions, depend on the same principle, and may be performed in the same manner, as in whole numbers.

ADDITION.

RULE.

Place the quantities to be added, in such order, that all that have the same local value may be directly under each other, and proceed exactly as in addition of whole numbers, separating the fractional from the integral part of their sum, by writing a decimal point between the place of tenths and the place of units.

Ex.—To add the quantities ·685 + 25·037 + ·85 + 3·25 + ·0045.

Here, placing the quantities as directed, we begin at the lowest place, writing under each column, the units in its amount, and adding the tens as so many units to the quantities in the next higher place; and finding the amount of the column of tenths to be 18, we write the 8-tenths under the column of tenths, and add the 10-tenths

$$\begin{array}{r} \cdot685 \\ 25\cdot037 \\ \cdot85 \\ 3\cdot25 \\ \cdot0045 \\ \hline 29\cdot8265 \end{array}$$

as 1 unit to the units of the whole numbers, from which we separate the fractional part of the amount by writing the decimal point between the 8 in the place of the tenths and the 9 in the place of units, and obtain the true sum, 29·8265.

EXAMPLES FOR PRACTICE.

1. Add 253·5 + 12·125 + 9·3456 + ·05 + ·0025 + 1·00075. *Ans.* 276·02385.

2. Add 17543·54 + 543·175 + 43·17543 + 3·756 + ·00007 + 25·875. *Ans.* 18159·5215.

3. Add ·875 + 9·0875 + 8·00875 + 1234·525 + ·00025 + 75864·34567. *Ans.* 77116·84217.

4. Add ·012 + ·00123 + ·5678 + 25·0065 + 125·03765 + 1756. *Ans.* 1906·62518.

5. Add ·123456 + 1·23456 + 12·3456 + 123·456 + 1234·56 + 12345·6 + 123456. *Ans.* 137173·319616.

6. Add 34567 + 3456·7 + 345·67 + 34·567 + 3·4567 + ·34567. *Ans.* 38407·73937.

SUBTRACTION.

RULE.

Place the smaller quantity under the greater, in the same order as in addition, and proceed exactly as in subtraction of whole numbers, separating the fractional from the integral part of the remainder, by writing a decimal point between the place of tenths and the place of units.

Ex.—To subtract 1·785 from 25·0374.

25·0374
1·785
23·2524

Here, beginning at the lowest place, and subtracting every figure in the lower line from that of the same local value in the upper, we find a cipher in the place of tenths in the upper line; we therefore take one of the five units, and expressing its value as 10-tenths in that place, make the subtraction and get the remainder, 2; and separating the fractional from the integral part of the remainder, by writing the decimal point between the 2 in the place of

tenths, and the 3 in the place of units, we obtain the true remainder, 23·2524.

EXAMPLES FOR PRACTICE.

(1.) From 12·5 take 5·7, and 123·456 — 75·98745.
Ans. 6·8 and 47·46855.

(2.) 957·999 — 578·125, and 25·665 – 18·76985.
Ans. 379·874 and 6·89515.

(3.) 17·1234 — ·789789, and 135·00075 – 5·000075.
Ans. 16·333611 and 130·000675.

(4.) 1 – ·876543211, and 1 — ·012345679.
Ans. ·123456789 and ·987654321.

(5.) 345 — ·999345 and ·000675 — ·00056789.
Ans. 344·000655 and ·00010711.

(6.) 100·001 — 99·999 and 1000 — 99·999999.
Ans. ·002 and 900·000001.

MULTIPLICATION.

1. Multiplication of decimal fractions is performed exactly in the same manner as multiplication of whole numbers; but as in decimal fractions the numerators only are written, the product thus obtained, will be the product of the numerators only.

2. Consequently, that this product of the numerators may express the product of the fractions, it must be made to consist of as many decimal places as there would be ciphers in the product of the denominators if they had been actually multiplied together.

3. Now, as the denominator of a decimal fraction is always the unit 1 with ciphers on its right hand, it is evident that the product of the denominators, will always contain just as many ciphers as are contained in both the denominators.

4. And as the numerator of a decimal fraction must always consist of as many places as there are ciphers in its denominator, it is equally evident that the product of the numerators, will always consist of as many places as are contained in both the numerators.

5. Hence we have only to multiply the numerators of the decimal fraction as if they were whole numbers, and to mark off from the right hand of the product, as many places as are contained in the multiplicand and in the multiplier together, and we shall obtain the true product of the fractions.

Thus to multiply $\cdot 7$ by $\cdot 5$; $7 \times 5 = 35$, is the product of the numerators, and $10 \times 10 = 100$, is the product of the denominators; and marking off two places from the right hand of the product of the numerators, 35, we have $\cdot 35 = \frac{35}{100}$, the true product of the fractions.

This may appear more clearly if we multiply the quantities after the manner of vulgar fractions; thus, $\cdot 7 = \frac{7}{10}$, and $\cdot 5 = \frac{5}{10}$; and $\frac{7}{10} \times \frac{5}{10} = \frac{7 \times 5}{10 \times 10} = \frac{35}{100} = \cdot 35$.

Also to multiply $\cdot 07$ by $\cdot 05$; $7 \times 5 = 35$, the product of the numerators, and $100 \times 100 = 10000$, the product of the denominators; consequently the product of the numerators must consist of four places; but as there are only two figures in this product, we add two ciphers to the left hand, and thus obtain $\cdot 0035 = \frac{35}{10000}$, the true product of the fractions.

For $\cdot 07 = \frac{7}{100}$, and $\cdot 05 = \frac{5}{100}$, and $\frac{7}{100} \times \frac{5}{100} = \frac{7 \times 5}{100 \times 100} = \frac{35}{10000} = \cdot 0035$;

It is evident that if the figures 35 were written immediately after the point, they would express $\frac{35}{100}$, which is 100 times greater than the true product; but by placing two ciphers on the left hand, they are removed two places farther to the right hand, and thus made 100 times less, and consequently express $\frac{35}{10000}$, which is the true product.

6. If the quantities to be multiplied are partly integral and partly fractional, as in the mixed number $7 \cdot 25$, we multiply exactly in the same manner as before without regarding the decimal point, and marking off from the right

hand of the product, as many decimal places as are contained both in the multiplicand and in the multiplier together, we obtain the true value of the fractional part, and also separate it from the integral part of the product.

Thus to multiply 7·25 by 3·5 :—

Here, multiplying exactly as in whole numbers, we obtain the product 25375; which contains the product of the whole numbers, and also of the fractions indiscriminately mixed; but by marking off from the right hand three places of decimals, the number contained in the multiplicand and multiplier together, we get the true product 25·375 = 25 units and 375 thousandths. Hence the

$$\begin{array}{r} 7{\cdot}25 \\ 3{\cdot}5 \\ \hline 3625 \\ 2175 \\ \hline 25{\cdot}375 \\ \hline \end{array}$$

RULE.

1. Place the multiplier under the multiplicand in the same order, and proceed exactly as in multiplication of whole numbers, without regarding either the decimal point or the particular local value of the quantities.
2. Mark off from the right hand of the product as many places of decimals as are equal to the number contained both in the multiplicand and in the multiplier.
3. If there be not so many figures in the product as the number of places to be marked off, place on the left hand of the product as many ciphers as will supply the deficiency.

EXAMPLES FOR PRACTICE.

Find the products of the following quantities :—

(1). ·2345 × ·0025 ; ·123 × ·00005 ; and 23·35 × 7·36.
Ans. ·00058625, ·00000615, and 171·856.

(2). 5763 × ·0075 ; 12·0075 × 9·345 ; and ·785 × 23·456.
Ans. 42·5475, 112·2100875, and 18·41296.

(3). ·17505 × 325 ; 123·999 × 25·6 ; and 315·45 × ·666.
Ans. 56·89125, 3174·3744, and 210·0897.

(4). 19·0007 × 3·005 ; and ·00700125 × ·009005.
Ans. 57·0971035, and ·0000630462562.

(5). 123·456789 × ·45; and ·5678 × ·08765.
Ans. 55·55555505, and ·04976767.

(6). 5·00012305 × 125; and ·0000075 × 1·025.
Ans. 625·01538125, and ·0000076875.

DIVISION.

1. Division of decimal fractions is performed exactly in the same manner as division of whole numbers; but as in decimal fractions the numerators only are written, the quotient thus found will be the quotient of the numerators only.

2. Consequently, that this quotient of the numerators may express the true quotient of the fractions, it must be made to consist of so many decimal places only, as there would be ciphers in the quotient of the denominators, if they had been actually divided.

3. Now as the denominator of a decimal fraction is always the unit 1 with ciphers on the right hand, it is evident that the quotient of the denominators will contain only as many ciphers as the denominator of the dividend has more than the denominator of the divisor.

4. And as the numerator of a decimal fraction must always consist of as many decimal places as there are ciphers in its denominator, it is equally evident that the quotient of the numerators must always consist of as many decimal places only, as the numerator of the dividend has more than the numerator of the divisor.

5. Hence we have only to divide the numerator of the dividend by the numerator of the divisor exactly as if they were whole numbers, and mark off in the quotient just as many decimal places as the dividend has more than the divisor, and we shall obtain the true quotient of the fractions.

Thus, to divide ·15875 by ·05; here $15875 \div 5 = 3175$, is the quotient of the numerators, and $100000 \div 100 = 1000$, is the quotient of the denominators; and as in this quotient there are three ciphers, we mark off three decimal places in the quotient of the numerators 3175, and obtain 3·175 the true quotient of the fractions.

Here it may be observed, that in the quotient 3·175

there are just as many decimal places as there are ciphers in its denominator 1000, and also just as many decimal places as the dividend ·15875 has more than the divisor ·05.

That this is the true quotient of the fractions, will be more evident if we divide these quantities in the manner of vulgar fractions; thus $\cdot 15875 = \frac{15875}{100000}$, and $\cdot 05 = \frac{5}{100}$; and $\frac{15875}{100000} \div \frac{5}{100} = \frac{3175}{1000} = 3 \cdot 175$, the same quotient as before.

6. If there are not so many decimal places in the dividend as there are in the divisor, ciphers may be annexed to the right hand of the dividend without making any alteration in its magnitude, till the number of places is at least equal to the number in the divisor; for we have shown that ·5, ·50, ·500, are all equal quantities, for $\cdot 5 = \frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$, &c., &c.

Thus to divide ·75 by ·0005; here $\cdot 75 = \cdot 7500$, and $7500 \div 5 = 1500$, the quotient of the numerators; and as the dividend has just as many decimal places as the divisor, we mark off no decimal places in the quotient of the numerators 1500, which is therefore a whole number and the true quotient of the fractions; for $\cdot 7500 \div \cdot 0005 = \frac{7500}{10000} \div \frac{5}{10000} = \frac{1500}{1} = 1500$.

7. If a whole number be divided by a decimal fraction, it is evident from the definition of division (see p. 29) that the quotient will be just as many times greater than the dividend as the divisor is less than a unit.

8. Consequently, if without annexing ciphers to the dividend with a decimal point, we obtain the quotient exactly without a remainder, we must, to get the true quotient, annex to the right hand of the quotient thus found, just as many ciphers as there are decimal places in the divisor.

Thus, to divide the whole number, 75, by the decimal fraction ·001; here $75 \div \cdot 001 = \frac{75}{1} \div \frac{1}{1000} = \frac{75}{1} \times \frac{1000}{1} = \frac{75000}{1} = 75000$ the true quotient, which consists of the

original dividend, 75, with three ciphers on the right hand, or just as many ciphers as there are decimal places in the divisor ·001, and is consequently 1000 times as great as the dividend, 75, or just as many times greater as the divisor is less than a unit.

9. When there are not so many figures in the quotient of the numerators as there should be decimal places in the quotient of the fractions, we must place as many ciphers on the left hand of the figures in the quotient as will make up the requisite number, before we write the decimal point.

Thus, to divide ·00375 by ·75. Here, $375 \div 75 = 5$, the quotient of the numerators; but as the dividend, ·00375, has three decimal places more than the divisor, ·75, there must be three decimal places in the quotient, 5. Therefore, to make up that number, we place two ciphers on the left hand of the figure 5, and obtain ·005, the true quotient of the fractions; for $\cdot00375 \div \cdot75 = \frac{375}{100000} \div \frac{75}{100} = \frac{5}{1000} = \cdot005$. Hence the

RULE.

1. Place the dividend and divisor, and proceed exactly as in division of whole numbers, not regarding the decimal point during the division; and mark off from the right hand of the quotient, just as many decimal places as the dividend has more than the divisor.

2. If there are not so many decimal places, or so many figures in the dividend as there are in the divisor, annex as many ciphers to the right hand as may be requisite, writing a decimal point if there be none already in the dividend.

3. If, after all the figures or places in the dividend have been divided, there be a remainder, more ciphers may be annexed, and the division continued to any extent.

4. If there are not so many figures in the quotient as may be sufficient to mark off the requisite number of decimal places, place as many ciphers

on the left hand of it as will make up the requisite number, before you write the decimal point.

Note.—In marking off the decimal places in the quotient, every cipher that has been used must be counted as a decimal place in the dividend; and every cipher that has not been used must be cancelled.

Ex. 1.—To divide 4·1875 by 1·25.

Here, dividing exactly as in division of whole numbers, we get the quotient, 335; and as there are in the dividend, 4·1875, two decimal places more than in the divisor, 1·25, we mark off two decimal places from the right hand of the quotient, 335, and obtain 3·35, the true quotient.

```
1·25 ) 4·1875 ( 3·35
       3 75
       ----
        437
        375
        ---
         625
         625
         ---
```

Ex. 2.—To divide 352·5 by 46·875.

Here, annexing two ciphers to the right hand of the dividend, to make the number of decimal places equal to the number in the divisor, we get the quotient, 7; but as there is a remainder, 24375, we annex another cipher, and, continuing the division, get the quotient, 5; and as we have still a remainder, 9375, we annex another, and get the quotient, 2, making the whole quotient 752, from which, counting all the ciphers used, we mark off two decimal places, and get 7·52, the true quotient.

```
46·875 ) 352·500 ( 7·52
         328 125
         -------
          243750
          234375
          ------
            93750
            93750
            -----
```

Ex. 3.—To divide 4·84375 by 387·5.

Here, dividing exactly as in division of whole numbers, we get the quotient, 125; and as there are five decimal places in the dividend, and only one in the divisor, there must be four decimal places in the quotient; but as there are only three figures in the quotient, 125, we place a cipher on the left hand of the quotient, 125, to make up the requisite number, and obtain ·0125, the true quotient.

```
387·5 ) 4·84375 ( ·0125
        3 875
        -----
         9687
         7750
         -----
         19375
         19375
         -----
```

EXAMPLES FOR PRACTICE.

Find the quotients of the following fractions:—

(1.) 234·70525 ÷ 64·25, and ·345678 ÷ ·002.
Ans. 3·653 and 172·839.

(2.) ·1886 ÷ ·25, and ·75445 ÷ ·00625.
Ans. ·7544 and 120·712.

(3.) 225 ÷ ·00025, and ·0025 ÷ ·0000025.
Ans. 900000 and 1000.

(4.) 675 ÷ ·00027, and ·00027 ÷ 675.
Ans. 2500000 and ·0000004.

(5.) 1·000875 ÷ ·125, and 125·4375 ÷ ·75.
Ans. 8·007 and 167·25.

(6.) ·0004375 ÷ 35, and 35 ÷ ·0004735.
Ans. ·0000125 and 80000.

REDUCTION.

1. Reduction of Decimals is either the reducing of a vulgar fraction to an equivalent decimal fraction; or the reducing of a whole number of a lower denomination, to an equivalent decimal fraction of a higher integer.

2. But this last is precisely the same thing; for every whole number of a lower denomination, is a vulgar fraction of the next higher integer; as 3 farthings are $\frac{3}{4}$ of a penny; 5 pence $\frac{5}{12}$ of a shilling; and 7 shillings $\frac{7}{20}$ of a £.

PROBLEM I.

To reduce a vulgar fraction to an equivalent decimal fraction.

1. As the denominator of every decimal fraction consists of the unit 1, with ciphers on the right hand, it is evident that if we multiply both terms of the vulgar fraction by such a number as will make its denominator either 10, or 100, or 1000, &c., we shall have the equivalent decimal fraction required.

Thus, to reduce the vulgar fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ to

equivalent decimal fractions: $\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10} = \cdot5$; $\frac{3}{4} = \frac{3\times25}{4\times25} = \frac{75}{100} = \cdot75$; and $\frac{5}{8} = \frac{5\times125}{8\times125} = \frac{625}{1000} = \cdot625$, which are respectively the equivalent decimal fractions required.

2. But 10, or 100, or 1000, &c., may not always be an exact multiple of the denominator of the given vulgar fraction, and consequently this method will not always apply; but as the numerators and denominators of equal fractions are always proportional quantities, the numerator of the decimal fraction may be always found by proportion in the given ratio of the denominators.

Thus, to reduce the same vulgar fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, to equivalent decimal fractions, we have in the first fraction, $\frac{1}{2}$, the given ratio of the denominators, $2 : 10$; hence $2 : 10 :: 1 : 5$, the numerator of the equivalent decimal fraction, $\frac{5}{10} = \cdot5$.

In the second fraction, $\frac{3}{4}$, we have the ratio of the denominators, $4 : 100$; hence $4 : 100 :: 3 : 75$, the numerator of the equivalent decimal fraction, $\frac{75}{100} = \cdot75$.

And in the third fraction, $\frac{5}{8}$, we have the ratio of the denominators, $8 : 1000$; hence $8 : 1000 :: 5 : 625$, the numerator of the equivalent decimal fraction, $\frac{625}{1000} = \cdot625$. Hence the

RULE.

> Annex with a decimal point as many ciphers as may be requisite, to the right hand of the numerator of the given vulgar fraction; and divide by its denominator, and the quotient will be the equivalent decimal fraction required.

Ex.—To reduce the vulgar fraction, $\frac{6}{125}$ to an equivavalent decimal fraction.

Here, annexing three ciphers to the right hand of the numerator, 6, and dividing by the denominator, 125, we get the quotient, 48; to which, prefixing a cipher on the left hand to mark off the requisite number of decimal places, we obtain ·048, the required decimal fraction, equal to the given vulgar fraction, $\frac{6}{125}$.

```
125 ) 6·000 ( ·048
      500
      ----
      1000
      1000
      ----
```

Ex. 2.—To reduce $\frac{5}{36}$ to an equivalent decimal fraction.

Here, placing two ciphers on the right hand of the numerator, 5, and dividing by the denominator, 36, we get the quotient, 13; but as we have the remainder, 32, we annex another cipher, and, continuing the division, get the quotient, 8, making the whole quotient 138, with the remainder 32.

```
36 ) 5·00 ( ·138
     36
     ---
     140
     108
     ---
      320
      288
      ---
       32
```

But as this remainder, 32, is the same as the preceding remainder, to whatever extent the division may be continued, the same remainder will perpetually recur; and its value, $\frac{32}{36} = \frac{8}{9}$, must be added to the figures, 138, making the entire quotient $\cdot 138\frac{8}{9}$, which is not a decimal fraction.

Hence it is evident that the magnitude of the vulgar fraction, $\frac{5}{36}$, cannot be expressed by a decimal fraction; for the decimal, ·138, wants $\frac{8}{9}$ of another thousandth part; and if, by continuing the division, we get the quotient, ·138888, this last decimal would want $\frac{8}{9}$ of another millionth part to express the true magnitude of the vulgar fraction, $\frac{5}{36}$.

When the same figure, as in this instance, or the same series of figures, as in other instances, perpetually recurs, these quantities are called repeating or circulating decimals; and they are expressed by writing a point over

the recurring figure, as $\dot{8}$, or over the first and last figures of the recurring series, as $\cdot\dot{1}2\dot{5}$.

A due investigation of this subject belongs rather to algebra than to arithmetic; therefore, when such repeating or circulating decimals occur, it will be necessary in all cases in which perfect accuracy is required, to substitute vulgar fractions in place of these quantities, which are only approximations.

EXAMPLES FOR PRACTICE.

1. Reduce the vulgar fractions, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$, to equivalent decimal fractions. *Ans.* ·5, ·25, and ·75.

2. Reduce $\frac{3}{8}$, $\frac{4}{25}$, and $\frac{7}{16}$, to equivalent decimal fractions. *Ans.* ·375, ·16, and ·4375.

3. Reduce $\frac{5}{12}$ and $\frac{576}{3072}$ to equivalent decimal fractions. *Ans.* $\cdot41\dot{6}$ and ·1875.

4. Reduce $\frac{15}{192}$ and $\frac{16}{125}$ to equivalent decimal fractions. *Ans.* ·78125 and ·128.

5. Reduce $\frac{6}{384}$ and $\frac{18}{1536}$ to equivalent decimal fractions. *Ans.* ·015625 and ·01171875.

6. Reduce $\frac{48}{768}$ and $\frac{84}{896}$ to equivalent decimal fractions. *Ans.* ·0625 and ·09375.

7. Reduce $\frac{7}{1792}$ and $\frac{747}{960}$ to equivalent decimal fractions. *Ans.* ·00390625 and ·778125.

8. Reduce $\frac{5}{12}$ of $\frac{8}{11}$, and $\frac{3}{8}$ of $\frac{11}{12}$ of $\frac{16}{25}$ to decimal fractions. *Ans.* $\cdot\dot{3}\dot{0}$ and ·22.

9. Reduce $\frac{11\frac{4}{5}}{25}$ and $\frac{1}{8000}$ of 15 to decimal fractions. *Ans.* ·472 and ·001875.

10. Reduce $\frac{7}{8}$ of $\frac{14}{35}$ of $\frac{8}{16\frac{2}{3}}$, and $\frac{2}{5}$ of $\frac{7}{12}$ of $\frac{11}{21}$ of $2\frac{1}{2}$ to decimal fractions. *Ans.* ·168 and $\cdot30\dot{5}$

PROBLEM II.

To reduce a quantity consisting of several denominations, to an equivalent decimal fraction of the highest integer.

1. Every whole number of a lower denomination, as we have already shown, is a vulgar fraction of the next higher integer; and if there be several denominations in the given quantity, they may be all included in one vulgar fraction.

For if we reduce the given quantity into its lowest denomination, we shall have the numerator; and the number of this lowest denomination contained in a unit of the required integer, will be the denominator of a vulgar fraction equivalent to the whole of the given quantity.

Thus, to reduce 15*s.* $6\frac{3}{4}$*d.* to the decimal fraction of a £.; we shall have 15*s.* $6\frac{3}{4}$*d.* = 747 farthings, the numerator; and 960, the number of farthings contained in a £. the denominator of the vulgar fraction $\frac{747}{960} =$ 15*s.* $6\frac{3}{4}$*d.*

Consequently, we have only to reduce this vulgar fraction, $\frac{747}{960}$, to an equivalent decimal fraction by Problem 1, and we shall have $\frac{747}{960} = \cdot 778125$, the equivalent decimal fraction required.

2. But if we write the several whole numbers in the given quantity under each other, and, beginning with the lowest denomination, divide each of them successively by such a number as will make a unit of the next higher denomination, placing the quotient on the right hand of the next higher whole number, the quotient of the last will be the decimal fraction required.

Thus, to reduce 15*s.* $6\frac{3}{4}$*d.* to the decimal fraction of a £.

Here placing the 6 pence under the 3 farthings, and the 15 shillings under the 6 pence, we divide the farthings by 4, and place the quotient, 75, on the right hand of the 6 pence; and dividing the 6 pence and ·75, or 6·75 by 12, we have the quotient, 5625, which we place on the right hand of the 15 shillings; and lastly, dividing 15·5625 shillings by 20, we get the quotient,

$$\begin{array}{r|l} 4 & 3 \\ 12 & 6\cdot 75 \\ 20 & 15\cdot 56250 \\ \hline & £\cdot 778125 \end{array}$$

·778125, which is the decimal fraction of the required integer, a £.

This is evidently nothing more than reduction of whole numbers; for as 4 farthings make 1 penny, 4-tenths of a farthing must make 1-tenth of a penny; and as 12 pence make 1 shilling, so 12-tenths of a penny must make 1-tenth of a shilling, &c. Hence the

RULE.

1. Reduce the given quantity into its lowest denomination, and annexing ciphers, divide by the number contained in a unit of the required integer, and the quotient will be the equivalent decimal fraction, or,
2. Write the several whole numbers in the given quantity under each other, beginning with the lowest denomination; and divide each of them successively by such a number as will make a unit of the next higher denomination, placing each quotient on the right hand of the next higher whole number, with a decimal point before it; and the last quotient will be the decimal fraction required.

EXAMPLES FOR PRACTICE.

1. Reduce 17*s.* and 17*s.* 6*d.* to equivalent decimal fractions of a £. *Ans.* ·85£., and ·875£.

2. Reduce 4*s.* 6¾*d.* and 3*s.* 10½*d.* to decimal fractions of a crown. *Ans.* ·9125 cr., and ·775 cr.

3. Reduce 2 qrs. 14 lb. and 3 qrs. 21 lb. to the decimal of a cwt. *Ans.* ·625 cwt., and ·9375 cwt.

4. Reduce 1 yd. 1 qr. 1 na. and 4 qrs. 2 na. to the decimal of a French ell. *Ans.* ·875 Fr. ell, and ·75 Fr. ell.

5. Reduce 3 fur. 35 po. 22 yds. to the decimal of a mile. *Ans.* ·485625.

6. Reduce 9 oz. 15 dwt. 18 grs. to the decimal of a lb. troy. *Ans.* ·815625.

7. Reduce 36 gal. 3 qt. 1 pt. to the decimal of a hhd. *Ans.* ·585317 hhd.

8. Reduce 21 dys. 18 hrs. to the decimal of a month. *Ans.* ·725.

9. Reduce $\frac{2}{15}$ of a guinea to the decimal of a moidore.
Ans. $\cdot 10\dot{3}70\dot{3}$ moid.

10. Reduce 3 ro. 14 po. to the decimal of an acre.
Ans. ·8375.

11. Reduce $\frac{2}{3}$ of $\frac{5}{8}$ of 7*s.* 6*d.* to the decimal of a £.
Ans. ·15625 £.

12. Reduce 12 hrs. 30 min. 30 sec. to the decimal of a week. *Ans.* ·074454365 we.

PROBLEM III.

To find the integral value of a decimal fraction in the several inferior denominations of its integer.

1. As fractions vary inversely as their integers, it is evident that in reducing the given decimal into its inferior denominations, the fraction will be made as many times greater as its integer is made less; and consequently in each of them will become an improper fraction.

We have, therefore, only to reduce these improper fractions into whole or mixed numbers, by marking off in each product the right number of decimal places, and we shall obtain the integral value of the given decimal fraction of all the inferior denominations of its integer.

Thus, to find the integral value of the decimal fraction ·778125£.

$$\begin{array}{rr} & \cdot 778125£. \\ & 20 \\ \hline s. & 15\cdot 562500 \\ & 12 \\ \hline d. & 6\cdot 750000 \\ & 4 \\ \hline f. & 3\cdot 000000 \end{array}$$

Here, reducing the parts of a £. into parts of a shilling, we get the product 15562500 parts of a shilling, from which, marking off 6 places of decimals, we get the integral value, 15*s.* 2nd. Reducing the remaining decimal, ·562500, into pence, we get the product, 6750000 parts of a penny, from which, marking off 6 places of decimals, we get the integral value, 6 pence; and lastly, reducing the remaining decimal, ·75000, into farthings, we get the product, 3000000 parts of a farthing; from which, marking off 6 places, we get the integral value, 3 farthings, leaving no remainder; and these together give 15*s.* $6\frac{3}{4}$*d.*, the whole integral value of the decimal fraction ·778125.

Note.—All the ciphers on the right hand of the several products might have been cancelled, as they occurred, without making any alteration in their value.

RULE.

Reduce the given decimal fraction successively into all the inferior denominations of its integer, and mark off in each of the products the proper number of decimal places.

EXAMPLES FOR PRACTICE.

1. Find the integral values of ·875£. and ·37 of a crown.
Ans. 17*s.* 6*d.* and 1*s.* 10*d.*

2. Find the integral values of ·425 guinea, and ·3785 moidore. *Ans.* 8*s.* 11$\frac{1}{10}$*d.*, and 10*s.* 2½*d.*

3. Find the value of ·76 cwt. and ·125 ton.
Ans. 3 qrs. 1 lb. 1 oz. 14 drs., and 2 cwt. 2 qrs.

4. Find the value of ·9875 English ell and ·375 of a French ell. *Ans.* 4 qrs. 3 na. and 2 qrs. 1 na.

5. Find the value of ·7895 mile.
Ans. 6 fur. 12 po. 3 yd. 1 ft. 6 in. 2 bar.

6. Find the value of ·3657 of an acre.
Ans. 1 r. 18 po. 15 yd. 4 ft. 56 in.

7. Find the value of ·3785 of 12*s.* 6*d.*
Ans. 4*s.* 8¾*d.*$\frac{1}{10}$.

8. Find the value of ·078654 of a year.
Ans. 28 da. 17 ho. 0 mi. 32 sec.

9. Find the value of ·12345 of a degree in geographical and in stat. miles. *Ans.* 7$\frac{4}{10}$ geo. mi., and 8$\frac{5}{10}$ stat. mi.

10. Find the value of ·7985 of a lb. troy.
Ans. 9 oz. 11 dwt. 15 grs.

PROPORTION.

Proportion in decimals is precisely the same as in whole numbers, except that when the terms consist of several denominations, instead of reducing them into their lowest denomination, we reduce them to decimal fractions of their highest integer.

Ex.—If 2 qrs. 14 lb. of sugar cost £1 „ 17*s.* 6*d.*, what must be paid for 1 cwt. 3 qrs. 21 lb.?

```
    28)14          28)21          12) 6
     4) 2·5         4) 3·75       20)17·5
                                                 £.
G. R. sugar ·625  :  1·9375   ::  1·875 : 5·8125=£5„ 16s. 3d.
                     1·875                        20
                    ------                       ----
                     96375               s. 16·2500
                    135625                      12
                   155000                      ---
                   19375                 d.  3·00
                  -------
          ·625)3·6328125(5·8125
               3125
               ----
               5078
               5000
               ----
                781
                625
                ----
                1562
                1250
                ----
                 3125
                 3125
                 ----
```

Here, instead of reducing the 2 qrs. 14 lb. of the first term into pounds, we reduce them to the decimal fraction of a cwt. = ·625; also the 3 qrs. 21 lb. of the second term into the decimal of a cwt. = ·9375; and in the third term we reduce the 17*s.* and 6*d.* to the decimal of a £. = ·875. No further rule is necessary.

Examples for Practice.

1. If 3 qrs. 3 na. of cloth cost 2*s.* 6*d.*, what will 15 English ells 3 qrs. cost at the same rate?

Ans. £2 „ 12*s.*

2. If 1 lb. 12 oz. of tea cost 8*s.* 9*d.*, what will be the value of 3 cwt. 2 qrs. 14 lb.? *Ans.* £101 „ 10*s.*

3. If when wheat is 6*s.* 3*d.* per bushel, the penny loaf weighs 8 oz. 7 drs., what will it weigh when wheat is 7*s.* 6*d.* per bushel? *Ans.* 7 oz. 0½ drs.

4. If 1½ ounces of silver cost 8*s.* 3*d.*, what will be the price of a tankard weighing 20 oz. 15 dwt. 18 grs.?

Ans. £5 „ 14*s.* 3¼*d.* $\frac{9}{16}$.

5. What will be the value of a pack of wool weighing 2 cwt. 3 qrs. 21 lb., at £1 „ 3*s*. 6*d*. per tod?

Ans. £27 „ 12*s*. 3*d*.

6. What will the tax upon £365 „ 17*s*. 6*d*. amount to, at 2*s*. 4½*d*. in the £.? *Ans*. £43 „ 8*s*. 11¼*d*.

7. If when the days are 12½ hours long, a field of 24 ac. 3 ro. 39 po. can be reaped, how much can be done by the same men in the same number of days when they are 15 ho. 48 mi. long?

Ans. 31 ac. 2 ro. 14 po. 22 yds. 2 ft.

8. If 3 qrs. 21 lb. of raisins cost 1 guinea, what must be paid for 3 casks, each weighing 1 cwt. 3 qrs. 14 lb.?

Ans. £6 „ 6*s*.

9. An estate of £3500 is assessed at 2*s*. 9¾*d*. in the £. upon $\frac{4}{5}$ of the rental, what will the rate produce?

Ans. £393 „ 15*s*.

10. What is the value of 3 cwt. 2 qrs. 17½ lb. of lead, at £25 „ 12*s*. 6*d*. per fother of 19½ cwt.?

Ans. £4 „ 16*s*. 1*d*. and $\frac{5}{10}$ fa.

11. How many hhds. of brandy, at £80 „ 17*s*. 6*d*. per hhd., can I have in exchange for 3¾ pipes of wine, at £121 „ 6*s*. 3*d*. per pipe? *Ans*. 5⅝ hhds.

12. If 51¾ Flem. ells cost 5½ guineas, what must be paid for 62$\frac{1}{10}$ English ells? *Ans*. £11 „ 11*s*.

DUODECIMALS.

1. Duodecimals are fractions of which the integer is divided into twelve equal parts or twelfths, and these twelfths in like manner are subdivided into twelfths of twelfths, &c. They are used by various artificers to ascertain the superficial contents of their work, which is valued by the square foot, square yard, or square rod, according to the nature of the work, or the particular custom of the trade.

2. The quantity of the work is measured by yards, feet, and inches of length, and also of breadth; and these dimensions being multiplied together, give the number of

square yards, square feet, square inches, &c., contained in the surface or superficial content.

3. The subdivisions of the inch do not take their name from their denominators, but from their relation to the integer, as first, second, or third subdivisions. Thus, the parts of an inch are called primes; parts of a prime, seconds; and the parts of a second, thirds; marked respectively ($'$), ($''$), ($'''$), as in the following

TABLE.

12 thirds ($'''$) ...	1 second	or 144 square thirds..	1 square second.
12 seconds ($''$) ..	1 prime	or 144 square seconds	1 square prime.
12 primes ($'$)....	1 inch	or 144 square primes .	1 square inch.
12 inches	1 foot	or 144 square inches .	1 square foot.
3 feet	1 yard	or 9 square feet ...	1 square yard.
10 feet	1 rod of slating	or 100 square feet ...	1 square rod of slating.
16¼ feet	1 rod of brickw.	or 272¼ square feet ...	1 square rod of brickwork.

4. Glaziers and masons generally charge for their flat work by the square foot; painters, plasterers, and paviers by the square yard; carpenters for partitioning and flooring, and slaters and tilers for roofing, by the square yard; and bricklayers by the square rod, allowing only one brick and a half in thickness, to which standard their work must be reduced before it is valued.

5. In duodecimals, the only thing requiring notice is the method of ascertaining the superficial content, by multiplying together or squaring the dimensions of length and breadth by what is called "Cross Multiplication," according to the following

RULE.

1. Place the dimensions of length and breadth in such order that those which are of the same denomination may stand under each other—feet under feet, inches under inches, primes under primes, &c.

2. Multiply the several denominations of the multiplicand, beginning with the lowest, by the highest denomination of the multiplier, carrying always to the product of the next higher as many units as there are twelves in the product of the de-

nomination multiplied, under which write the remainder.

3. In the same manner, multiply the several denominations of the multiplicand by the next lower denomination, and successively by all the inferior denominations of the multiplier, observing to place the lowest product in every succeeding line one place to the right hand of the lowest product in the preceding line; or as many places to the right hand of it as the multiplier you are using, is distant from the preceding multiplier.

4. Continue this process till the whole of the multiplier has been used, and the several lines of products, added together, will be the whole product or superficial content required.*

Ex.—To multiply 16 feet, 7 inches, 9 primes, by 5 feet, 6 inches, 0 primes, 7 seconds.

ft.	in.	'			
16	7	9			
5	6	0	7'		
83	2	9			
8	3	10	6''		
		9	8	6'''	3''''
91	7	5	2	6	3

Here, placing the several denominations of the multiplier under those of the same name in the multiplicand, and first multiplying the 9 primes in the multiplicand, by the 5 feet in the multiplier, we get the product, 45, in which are 3 twelves, and the remainder, 9. We therefore write 9 under the primes in the multiplicand, and add the 3 twelves as 3 units to the product of the inches, and so on throughout. In the second line, multiplying the 9 primes of the multiplicand by the 6 inches in the multiplier, we get the product, 54, in which are 4 twelves, and the remainder, 6; but as the multiplier in this product is one place to the right hand of the former multiplier, 5, we write the remainder, 6, one place to the right hand of the lowest product in the line above. In the third line, we multiply the 9 primes in the multiplicand by the 7 fourths of the multiplier, and obtain the product, 63, in which are 5 twelves, and the remainder, 3; but as the multiplier, 7, is two places to the right of the preceding multiplier, 6, we place the remainder, 3, two places to the right hand of the lowest product in the line above. And, lastly, adding together the several lines of products, we

get the whole product, or superficial content, of the dimensions of length and breadth.

That these several products are justly arranged according to their true values, may be thus demonstrated: for as a foot is here the integer, in multiplying the 9 primes by the 5 feet, we have 9 primes $= \frac{9}{144}$ of a foot, and 5 feet $= \frac{5}{1}$ ft. Hence $\frac{9}{144} \times \frac{5}{1} = \frac{45}{144}$ of a foot; but $\frac{45}{144} = \frac{36 \times 9}{144}$, and $\frac{36}{144} = \frac{3}{12}$ ft. $= 3$ inches; also, $\frac{9}{144}$ feet $= 9$ primes: consequently the 9 primes are properly written in the place of primes, and the 3 inches added to the product of the inches.

Also, in the second line we have the 9 primes $= \frac{9}{144}$ of a foot, multiplied by 6 inches $= \frac{6}{12}$ ft. Hence $\frac{9}{144} \times \frac{6}{12} = \frac{54}{1728}$ of a foot, but $\frac{54}{1728} = \frac{48+6}{1728}$, and $\frac{48}{1728} = \frac{4}{144}$ ft. $= 4$ primes; also, $\frac{6}{1728} = 6$ seconds, which are, properly written, one place to the right hand of the primes in the line above, in which places they have respectively their true value, and the same reasoning will apply to all the other products.

EXAMPLES FOR PRACTICE.

Multiply the following dimensions of length and breadth, and find the superficial contents:—

(1.) 9 ft. 5 in. × 6 ft. 7 in. *Ans.* 61 ft. 11 in. 11′.

(2.) 16 ft. 5 in. 3′ × 7 ft. 5 in. 8′.
Ans. 122 ft. 9 in. 10′ 9″.

(3.) 15 ft. 7 in. 5′ × 3 ft. 0 in. 9′.
Ans. 47 ft. 9 in. 11′ 6″ 9‴.

(4.) 11 in. 8′ 9″ × 9 in. 7′ 6″.
Ans. 9 ft. 4 in. 10′ 8″ 7‴ 6⁗.

(5.) 35 ft. 6 in. 3′ × 12 ft. 0 in. 5′.
Ans. 47 yds. 4 ft. 5 in. 9′ 7″ 3‴.

(6.) 17 ft. 0 in. 6′ × 5 ft. 0 in. 7′.
Ans. 86 ft. 0 in. 5′ 3″ 6‴.

7. What will the paving of an area, 88 ft. 9 in. long, and 48 ft. 11 in. broad, cost at $7\frac{3}{4}d.$ per square yard? *Ans.* £15 ,, 11*s.* $6\frac{1}{4}d.$ $\frac{83}{108}$.

8. What must be paid for the ceiling of a room, 72 ft. 9 in. in length, and 24 ft. 3 in. in breadth, at 3*s.* 10*d.* per square yard? *Ans.* £41 ,, 16*s.* $7\frac{1}{2}$.

9. What will be the cost of a marble slab, 11 ft. 2 in. long, and 3 ft. 8 in. wide, at 7*s.* 6*d.* per foot square? *Ans.* £15 ,, 15*s.* 5*d.*

10. What will be the cost of wainscoting a room, 24 ft. 9 in. long, and 19 ft. 6 in. wide, at 13*s.* 4*d.* per yard square? *Ans.* £35 ,, 15*s.*

11. What must be paid for the slating of a house, 64 ft. 6 in. long, 32 ft. 8 in. broad on the flat, the roof being of a true pitch, or each slant $\frac{3}{4}$ of the breadth, and the eave-board projecting 18 in. on each side, at £1 ,, 13*s.* 4*d.* per square of 100 feet? *Ans.* £55 ,, 18*s.*

12. What must be paid for the building of a garden wall, 130 yards long, 6 feet high, and $2\frac{1}{2}$ bricks thick, at 10 guineas per square rod? *Ans.* £150 ,, 8*s.* $3\frac{21}{121}d.$

13. What will be the expense of flooring a room which is 75 ft. 9 in. long, and 35 ft. 8 in. wide, at 16*s.* 8*d.* per square? *Ans.* £22 ,, 10*s.* $3\frac{1}{2}d.$

14. What must be paid for glazing three tiers of windows, having 3 windows in each tier, the windows in the first tier 12 ft. 6 in.; in the second, 9 ft. 4 in.; and the third, 6 ft. 2 in. in height; and the common width of all 4 ft. 6 in., at 15*d.* per square foot? *Ans.* £23 ,, 12*s.* 6*d.*

15. The walls of a rectangular hall, 120 ft. long, 47 ft. 9 in. wide, and 25 ft. 6 in. high, are to be stuccoed at 15*d.* per square yard, the ceiling to be plastered at $10\frac{1}{2}d.$, and afterwards painted in device at 2*s.* 6*d.* per square yard, and a skirting-board of oak, 18 inches deep, is to be placed round the walls at $10\frac{1}{2}d.$ per foot. What will the whole of this work cost? *Ans.* £188 ,, 17*s.* $3\frac{7}{8}d.$

PRACTICE.

1. This Rule derives its name from its general use as a short method of finding the value of any quantity of goods or number of articles when the value of one is given; and is only an abridgment of Proportion, in which the first term being always 1, the whole process resolves itself into a simple act of multiplication.

2. Its brevity arises chiefly from considering the given price as a fraction of a unit of the next higher integer; for in multiplying by a fraction, we have only to take for the product such a part or parts of the multiplicand as the multiplier is of a unit, and we obtain the product more expeditiously.

3. If we assume a unit of the next higher integer as the price, it is evident that the number of articles will itself express their value at this assumed price; and if we take such part or parts of this value as the given price is of the assumed price, we shall have the true value at the given price.

4. But in taking these parts, it is to be observed that if the given price be not itself an exact part of a unit of the next higher integer, it must be divided into portions which will be exact parts of the integer or of each other; and the several values corresponding to these parts, added together, will be the whole value required.

Thus, to find the value of any number of articles at 17*s.* 6*d.* each, 17*s.* 6*d.* $= \frac{7}{8}$ of a £., which is not an exact part of the next higher integer, £1; but $\frac{7}{8} = \frac{4+2+1}{8}$ £., and $\frac{4}{8} = \frac{1}{2}$ £. $= 10s.$; also $\frac{2}{8}$ £. $= \frac{1}{4}$ £. $= 5s.$, and $\frac{1}{8}$ £. $= 2s.\ 6d.$; consequently, we divide the given price, 17*s.* 6*d.*, into the three parts, 10*s.*, 5*s.*, and 2*s.* 6*d.*, each of which is an exact part of a £.

Hence, if we take $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of the given number of

articles, which, at the assumed price, £1, express their own value, and add together these several parts, their sum will be the true value of the number of articles at the given price, 17*s*. 6*d*.

Ex.—What is the value of 1234 yards of cambric at 17*s*. 6*d*. per yard?

Here, the value of the 1234 yards at the assumed price, £1, is £1234; therefore, for their value at 10*s*., which is just $\frac{1}{2}$ £., we take one-half this value and get £617, their value at 10*s*.; for their value at 5*s*, which is $\frac{1}{4}$ of a £., we take $\frac{1}{4}$ of the value of £1, or what is the same thing, $\frac{1}{2}$ of their value at 10*s*., and get £308 10*s*., their value at 5*s*.; in the same manner, taking $\frac{1}{8}$ of the value of £1, or what is the same, $\frac{1}{2}$ of their value at 5*s*., we get £154 „ 5*s*., their value at 2*s*. 6*d*.; and lastly, adding together these several values, we get their sum, £1079 „ 15*s*., the value at 17*s*. 6*d*. each.

s.	*d*.		£.	*s*.
			1234	0
10	0	$= \frac{1}{2} =$	617	0
5	0	$= \frac{1}{2} =$	308	10
2	6	$= \frac{1}{2} =$	154	5
			£1079	15

5. Had the price been £1 „ 17*s*. 6*d*. each, the only difference would have been, that instead of drawing a line under the given number, 1234, we should have added the £1234, their value at £1 each, with the values obtained for the several parts of the integer; or had the given price been £5 „ 17*s*. 6*d*., we should have multiplied their value at £1 by 5, and then have added as before.

6. When there is a fractional part in the given number of articles, as, for instance 1234½; after having found, as before, the value of the whole number, 1234, we add to it such part or parts of the given price as the fractional part is of a unit.

7. When the given number of articles or quantity of goods consists of several denominations, as, for instance, 36 cwt. 2 qrs. 17½ lb., we multiply the given price by the number of the highest denomination in order to find their

value; and to find the values of the several numbers of the inferior denominations, we take such part or parts of the given price as each of them are respectively parts of an integer in the highest denomination, or of each other.

Ex.—To find the value of 36 cwt. 2 qrs. $17\frac{1}{2}$ lb. of sugar at £1 „ 17*s*. 6*d*. per cwt.

Here, multiplying the given price, £1 „ 17*s*. 6*d*., by $6 \times 6 = 36$, the number in the highest denomination, cwts., we get their value, £67 „ 10*s*.; and as 2 qrs. are $\frac{1}{2}$ of a cwt., we take $\frac{1}{2}$ of the given price and get their value, 18*s*. 9*d*. Now, separating the $17\frac{1}{2}$ lb. into the two parts, 14 lb. and $3\frac{1}{2}$ lb., we find the 14 lb. = $\frac{1}{8}$ of a cwt. or $\frac{1}{4}$ of 2 qrs., and taking $\frac{1}{4}$ of 18*s*. 9*d*., the value of 2 qrs., we get 4*s*. $8\frac{1}{4}d$., the value of 14 lb.; and lastly, as $3\frac{1}{2}$ lb. $= \frac{1}{4}$ of 14 lb., we take $\frac{1}{4}$ of 4*s*. $8\frac{1}{4}d$., the value of 14 lb., and get 1*s*. 2*d*., the value of $3\frac{1}{2}$ lb., and now, adding together these several values, we get their sum, £68 „ 14*s*. $7\frac{1}{4}d$., the value of the whole quantity, 36 cwt. 2 qrs. $17\frac{1}{2}$ lb.

	£.	s.	d.
	1	17	6
			6
	11	5	0
			6
	67	10	0
2 qr. $= \frac{1}{2} =$		18	9
14 lb. $= \frac{1}{4} =$		4	$8\frac{1}{4}$
$3\frac{1}{2}$ lb. $= \frac{1}{4} =$		1	2
	£68	14	$7\frac{1}{4}$

It will be unnecessary to divide the subject into "Cases," and to multiply rules for their particular solution; the student who is well acquainted with proportion and with fractions will, by the exercise of his judgment, find in every case the readiest method of proceeding.

By way of proof, and as affording in many instances, a shorter solution, he may work the questions by Vulgar or Decimal Fractions, by reducing the given price to a fraction, and multiplying it by the given number of articles; as, for instance, the first example, 1234 at 17*s*. 6*d*. each: $1234 = \frac{1234}{1}$ and 17*s*. 6*d*. $= \frac{17\frac{1}{2}}{20} = \frac{35}{40} = \frac{7}{8}$£.; and $\frac{1234}{1} \times \frac{7}{8} = \frac{8638}{8}$£. $= 1079\frac{6}{8} =$ £$1079\frac{3}{4}$ = £1079 „ 15*s*.

EXAMPLES FOR PRACTICE.

(1.) 6142 oz. at $\frac{1}{4}d.$ per oz.; 1461 at $\frac{1}{2}d.$; and 4764 at $\frac{3}{4}d.$

Ans. £6 „ 7*s.* 11½*d.*; £3 „ 0*s.* 10½*d.*; and £14 „ 17*s.* 9*d.*

(2. 3165 at 1¼*d.*; 7630 at 2½*d.*; and 5276 at 3¾*d.*

Ans. £17 „ 6*s.* 4¼*d.*; £79 „ 9*s.* 7*d.*; and £82 „ 8*s.* 9*d.*

(3.) 7654 at 4¼*d.*; 8073 at 5½*d.*; and 2675 at 6¾*d.*

Ans. £135 „ 10*s.* 9½*d.*; £185 „ 0*s.* 1½*d.*; and £75 „ 4*s.* 8¼*d.*

(4.) 7462 at 9¾*d.*; 1264 at 10½*d.*; and 5176 at 11¾*d.*

Ans. £303 „ 2*s.* 10½*d.*; £55 „ 6*s.*; and £253 „ 8*s.* 2*d.*

(5.) 6241 at 1*s.* 1½*d.*; 7692 at 1*s.* 3*d.*; and 2645 at 1*s.* 9¾*d.*

Ans. £351 „ 1*s.* 1½*d.*; £480 „ 15*s.*; and £239 „ 14*s.* 0¾*d.*

(6.) 4286½ at 3*s.* 4*d.*; 1267 at 4*s.* 9½*d.*; and 1625¾ at 6*s.* 8*d.*

Ans. £714 „ 8*s.* 4*d.*; £303 „ 11*s.* 0½*d.*; and £541 „ 18*s.* 4*d.*

(7.) 6243 at 7*s.* 9¾*d.*; 1235 at 9*s.* 11½*d.*; and 1264 at 11*s.* 8*d.*

Ans. £2438 „ 13*s.* 5¼*d.*; £613 „ 12*s.* 9¼*d.*; and £737 „ 6*s.* 8*d.*

(8.) 6476 at 13*s.* 4*d.*; 1678½ at 15*s.* 10*d.*; and 2647 at 17*s.* 9¾*d.*

Ans. £4317 „ 6*s.* 8*d.*; £1328 „ 16*s.* 3*d.*; and £2357 „ 9*s.* 8¼*d.*

(9.) 1234 at 19*s.* 11¾*d.*; 6274 at £1 „ 10*s.* 6*d.*; and 1687 at £5 „ 13*s.* 4*d.*

Ans. £1232 „ 14*s.* 3½*d.*; £9567 „ 17*s.*; and £9559 „ 13*s.* 4*d.*

10. What is the value of 12 oz. 14 dwts. 16 grs. of gold at £3 „ 17*s.* 1½*d.* per ounce? *Ans.* £49 „ 2*s.* 0½*d.*

11. What is the value of 15 oz. 15 dwts. 10 grs. of silver at 5*s.* 7½*d.* per ounce? *Ans.* £4 „ 8*s.* 8½*d.* $\frac{1}{8}f.$

12. What is the value of 5 hhds. 42 gals. of brandy at £75 „ 16*s.* 8*d.* per hhd.? *Ans.* £429 „ 14*s.* 5¼*d.* ¼*f.*

13. What is the value of 35 cwt. 2 qrs. 10½ lb. of tobacco, at £25 „ 12*s.* 6*d.* per cwt.? *Ans.* £912 „ 1*s.* 9½*d.*

14. What is the value of 16 Fr. ells 4 qrs. 3 na. of velvet, at 17*s.* 6¼*d.* per ell? *Ans.* £14 „ 14*s.* 10¼*d.*

15. What is the value of 45 qrs. 7 bus. 3 pecks of wheat, at £3 „ 7*s*. 10*d*. per qr.? *Ans*. £155 „ 18*s*. 2½*d*.

16. What is the value of 21 tons 19 cwt. 3 qrs. 21 lb. of coal, at £1 „ 17*s*. 9¾*d*. per ton? *Ans*. £41 „ 11*s*. 9¼*d*. ½ *f*.

TARE AND TRET

Are certain allowances made to the purchasers of goods for packages, waste, &c., either of so much on the whole weight, or of so much per cwt., &c., the amount of which being deducted from the whole or gross weight, leaves the neat weight or quantity charged to the purchaser at the given price.

1. Tare is an allowance for the weight of the package containing the goods.

2. Tret is an allowance for waste or damage of 4 lb. in every 104 lb., or $\frac{1}{26}$th part of the quantity which remains after the tare has been deducted.

3. Cloff is an allowance for loss of weight of 2 lb. in every 3 cwt., or $\frac{1}{168}$th part of the quantity that remains after the Tare and Tret have been deducted.

4. Suttle is the quantity that remains after any of these allowances have been deducted, and from which some further deduction is to be made.

5. The neat weight, or net, is the quantity that remains after all the allowances have been deducted from the gross weight.

6. The method of finding the amounts of these several allowances must be left to the discretion of the student; and as there is nothing peculiar either in the principle or detail of this rule, it will be sufficient to give only a detailed example.

Ex.—What is the neat weight of 7 hhds. of sugar, weighing each 3 cwt. 2 qrs. 14 lb. gross; Tare, 7 lb. per cwt., Tret and Cloff as usual?

Here, multiply the weight of 1 hhd. by 7; we get the whole gross weight, 25 cwt. 1 qr. 14 lb., from which we deduct the tare $= \frac{1}{16}$ of the gross, leaving 23 cwt. 3 qr. 5 lb., which, being subject to further deduction, we call suttle. From this we deduct the tret, $\frac{1}{26} =$ 3 qr. 18 lb.,

	cwt.	qr.	lb.	
	3	2	14	
			7	
	25	1	14	gross.
$7 = \frac{1}{16} =$	1	2	9	tare.
	23	3	5	suttle.
$\frac{1}{26}$	0	3	18	tret.
	22	3	15	suttle.
$\frac{1}{168}$			15	cloff.
	22	3	0	net.

and get the remainder, 22 cwt. 3 qr. 15 lb., which, being still subject to further deduction, we call suttle; and lastly, from this we deduct the cloff $= \frac{1}{168} =$ 15 lb., and get the remainder, 22 cwt. 3 qr. 0 lb., which, as nothing further is to be deducted from it, is the net, or the neat weight of the 7 hhds.

EXAMPLES FOR PRACTICE.

1. What is the neat weight of 36 barrels of potash, together weighing 120 cwt. 0 qr. 17 lb., allowing 2 cwt. 3 qr. 19 lb. tare one the whole? *Ans.* 117 cwt. 0 qr. 26 lb.

2. What is the neat weight of 15 hhds. of tobacco, weighing each 3 cwt. 2 qr. 14 lb., allowing on the whole 3 cwt. 1 qr. 11 lb. for tare? *Ans.* 51 cwt. 0 qr. 3 lb.

3. What is the neat weight of 17 bags of pepper, weighing each 2 cwt. 1 qr. 9 lb., allowing 14 lb. per bag for tare? *Ans.* 37 cwt. 1 qr. 27 lb.

4. What is the neat weight of 25 boxes of soap, weighing each 1 cwt. 3 qrs. 12 lb., allowing tare at the rate of 10 lb. per cwt.? *Ans.* 42 cwt. 1 qr. 5 lb.

5. Required, the neat weight of 9 barrels of molasses, weighing together 116 cwt. 2 qr. 24 lb. Tare, $17\frac{1}{2}$ lb. per cwt., and tret as usual. *Ans.* 99 cwt. 1 qr. $25\frac{1}{2}$ lb.

6. What is the neat weight of 25 hhds. of sugar, weighing each 2 cwt. 3 qr. 18 lb., allowing 12 lb. per cwt. for tare, and tret and cloff as usual? *Ans.* 62 cwt. 1 qr. 26 lb.

7. What is the neat weight of 3 hhds. of tobacco,

weighing as follows:—No. 1, 3 cwt. 2 qr. 19 lb.; No. 2, 4 cwt. 0 qr. 17 lb.; and No. 3, 5 cwt. 1 qr. Tare, 9 lb. per cwt., and tret and cloff as usual?

Ans. 11 cwt. 2 qr. 1 lb.

8. What is the neat weight of 7 casks of flour, weighing each 2 cwt. 1 qr. 19 lb. Tare, $12\frac{1}{2}$ lb. per cwt., and tret 2 lb. per cask? *Ans.* 14 cwt. 3 qr. 21 lb.

9. What is the neat weight of 13 casks of raisins, weighing each $3\frac{1}{2}$ cwt. Tare, 10 lb. per cwt., tret, 3 lb. per cask, and cloff as usual? *Ans.* 40 cwt. 3 qr. 11 lb.

10. What is the value of the neat weight of 12 hhds. of sugar, weighing together 3084 lb. Tare, 14 lb. per cwt., and tret and cloff as usual, at $7\frac{1}{2}$*d.* per lb.?

Ans. £80 „ 13*s.* $1\frac{1}{2}$*d.*

11. What is the value of the neat weight of 17 bags of coffee, weighing each 3 cwt. 2 qr. $17\frac{1}{2}$ lb. Tare, 16 lb. per cwt., tret as usual, and cloff 3 lb. per bag, at the rate of £13 10*s.* per cwt.? *Ans.* £685 „ 8*s.* 6*d.*

12. What is the neat weight of 9 hhds. of molasses, weighing each 4 cwt. 0 qrs. $10\frac{2}{3}$ lb.; tare, $17\frac{1}{2}$ lb. per cwt., and tret and cloff as usual; and what is the value of it at £3 „ 12*s.* 4*d.* per cwt.?

Ans. Neat weight, 30 cwt. 1 qr. 26 lb.; and value, £110 „ 4*s.* $10\frac{1}{2}$*d.*

INTEREST.

1. Interest is an allowance paid for the loan or use of a sum of money for any given time, and which by law must not exceed £5 for the loan or use of £100 for one year.

2. When the interest is regularly paid as it becomes due, it is called Simple Interest; but when, instead of being paid, it is added to the principal, and with it put to interest, it is called Compound Interest.

At present we confine ourselves to Simple Interest, in treating of which, it may be necessary to observe, that

The sum of money put out to use is called the principal;

The sum paid for the use of the principal, for one year, is called the rate per cent.;

The number of years, months, or days, for which the principal is employed, is called the time;

The sum of money paid for the use of the principal, at the given rate, and for the given time, is called the interest; and

The sum arising from the addition of the interest to the principal, is called the amount.

Thus, if £100 be put out to use at 5 per cent. for one year, £100 is the principal, £5 is the rate per cent., one year is the time, £5 is the interest, and £105 is the amount.

3. If we examine the relation which these several elements have to each other, as variable and dependent quantities in the questions in which they occur, we shall find that—

1. The principals will always vary directly as the interests, inversely as the rates per cent., and also inversely as the times.

2. The rates per cent. will vary directly as the interests, inversely as the principals, and also inversely as the times.

3. The times will vary directly as the interests, inversely as the principals, and also inversely as the rates per cent.; and

4. The interests will vary directly as the principals,

directly as the rates per cent., and also directly as the times.

Hence it is obvious that Proportion affords an easy and ample solution of all possible questions in Simple Interest.

The rule usually given, viz., to multiply the principal by the rate per cent., and also by the time, and to divide their product by 100, is nothing more than finding the fourth term of a proportion, of which the first term is always 100.

For as the interest varies directly as the principals, and also as the times, the antecedent of the ratio of principals will always be 100, and the antecedent of the ratio of times always 1; consequently, $100 \times 1 = 100$ will always be the antecedent of the ratio compounded of these.

But this rule extends only to the finding of the interest, whereas Proportion embraces every possible variety. We shall, therefore, subjoin only some detailed examples, and leave the student to the exercise of his own judgment.

It may be well to remind him, that in every question in Simple Interest, it is always supposed that £100 in 1 year, at 5 per cent. gains £5 interest. It may be also well to observe, that when there is any number of days that make no exact part of a year, the interest for those days must be found separately, and added to that for the years and months.

Ex. 1.—What principal will gain £35 interest in 5 years, at the rate of 4 per cent. per annum?

$$\begin{array}{lrcl}
 & & & 7 \\
\text{Giv. ra. int. ..} & \cancel{5} & : & \cancel{35} \\
\text{Inv. ra. time .} & \cancel{5} & : & 1 \\
\text{Inv. ra. rates .} & 4 & : & \cancel{5} \\
\hline
\text{Comp. ratio ..} & 4 & : & 7 :: \pounds 100 : \pounds 175 \text{ principal.}
\end{array}$$

Ex. 2.—At what rate per cent. will £175 gain £35 interest in 5 years?

$$\begin{array}{lrcl}
\text{Giv. ra. int. ..} & \cancel{5} & : & \cancel{35} \\
 & 5 & & 4 \\
\text{Inv. ra. prin..} & \cancel{175} & : & \cancel{100} \\
\text{Inv. ra. time .} & \cancel{5} & : & 1 \\
\hline
\text{Comp. ratio ..} & 5 & : & 4 :: \pounds 5 : \pounds 4 = \text{rate per cent.}
\end{array}$$

Ex. 3.—In what time will £175 gain £35 interest, at the rate of 4 per cent.?

Giv. ra. int. . .	$\cancel{5}$ 5	:	$\cancel{35}$ 25
Inv. ra. prin. .	$\cancel{175}$	:	$\cancel{100}$
Inv. ra. rates .	$\cancel{4}$	:	$\cancel{5}$

Comp. ratio . . 5 : 25 : : 1 yr. : 5 yrs. = time.

Ex. 4.—What will be the interest of 175 for 5 years, at the rate of 4 per cent.?

Giv. ra. prin. .	4 $\cancel{100}$	:	7 $\cancel{175}$
Giv. ra. time .	1	:	$\cancel{5}$
Giv. ra. rates.	$\cancel{5}$	:	$\cancel{4}$

Comp. ratio . . 1 : 7 : : £5 : £35 = interest.

EXAMPLES FOR PRACTICE.

1. What is the interest of £375 „ 7*s*. 6*d*. for 1 year, at 5 per cent.? *Ans*. £18 „ 15*s*. 4½*d*.

2. What is the interest of 125 guineas for 2½ years, at 4 per cent.? *Ans*. £13 „ 2*s*. 6*d*.

3. What is the amount of £350 for 3½ years, at 3½ per cent.? *Ans*. £392 „ 17*s*. 6*d*.

4. In what time will 125 guineas gain 25 guineas, at 4 per cent.? *Ans*. 5 years.

5. What is the interest of £735 „ 17*s*. 6*d*. for 3 years 9 months, at 4½ per cent.? *Ans*. £124 „ 3*s*. 6¾*d*.

6. What is the amount of £1500 for 2 years and 8 months, at 5 per cent.? *Ans*. £1700.

7. What is the interest of £347 „ 15*s*. for 3 years and 52 days, at 4¼ per cent.? *Ans*. £51 „ 18*s*. 3¼*d*.

8. At what rate per cent. will £250 gain £35 interest in 4 years? *Ans*. 3½ per cent.

9. What is the interest due upon an India bond of £750, for 75 days, at 3½ per cent.? *Ans*. £5 „ 7*s*. 10½*d*.

10. What is the amount of £115 for 20 years at the rate of 5 per cent.? *Ans*. £230.

11. What will be the interest on an exchequer bill of £500, from Sept. 30th, 1843, to June 24th, 1844, at $3\frac{3}{4}$ per cent.? *Ans.* £13 „ 15*s*. 4*d*.

12. What principal will gain £75 interest in 5 years at 3 per cent.? *Ans.* £500.

13. A gentleman left his niece, who was 17 years, 9 months, 15 days old, £3215, to be paid with interest at $4\frac{1}{2}$ per cent. when she came of age; what will she have to receive? *Ans.* £3679 „ 3*s*. $3\frac{3}{4}$*d*.

14. A banker advanced £500 on the 1st of Jan., £750 on the 1st of March, and £1050 on the 20th of June, 1843; what will be due to him on the 1st of Jan., 1844, allowing interest at 5 per cent.? *Ans.* £2383 „ 19*s*. $0\frac{1}{2}$*d*.

15. If the interest of £300 at 5 per cent. was £150, for what time was it forborne? *Ans.* 10 years.

16. At what rate per cent. will the interest of £750 amount to £187 „ 10*s*. in 5 years? *Ans.* 5 per cent.

17. In what time will the interest on £250 amount to £125 at 4 per cent.? *Ans.* $12\frac{1}{2}$ years.

18. What will be the amount of £300 at 5 per cent., £500 at 4 per cent., and £750 at $2\frac{1}{2}$ per cent., in 5 years? *Ans.* £1818 „ 15*s*.

19. In what time will a sum of money double itself at the rate of 4 per cent.? *Ans.* 25 years.

20. At what rate per cent. will a sum of money at interest double itself in 40 years. *Ans.* $2\frac{1}{2}$ per cent.

COMMISSION, BROKERAGE, INSURANCE, PURCHASING OF STOCK, &c.

1. All these Rules are merely the names of the various purposes to which the principles of Proportion are applied to practical use in the transactions of business; they are calculations of certain allowances, of so much per cent., and may be obtained by Proportion or by Practice, according to the discretion of the student; it will therefore only be recuisite to give some

EXAMPLES FOR PRACTICE.

1. What will the commission upon £250 amount to, at $2\frac{1}{2}$ per cent. *Ans.* £6 „ 5*s.*

2. What will be the commission on £1750 „ 10*s.* 6*d.* at $1\frac{3}{8}$ per cent.? *Ans.* £24 „ 1*s.* $4\frac{1}{2}$*d.*

3. What must I allow my agents for selling goods on my account to the amount of £315 „ 17*s.* 6*d.*, on commission at $2\frac{1}{4}$ per cent.? *Ans.* £7 „ 2*s.* $1\frac{1}{2}$*d.*

4. What will be the commission on £1350 „ 15*s.* at $3\frac{5}{8}$ per cent.? *Ans.* £48 „ 19*s.* $3\frac{1}{2}$*d.*

5. If I allow a broker $\frac{7}{8}$ per cent., what must I pay him for selling £1500 stock? *Ans.* £13 „ 2*s.* 6*d.*

6. What must I pay for the purchase of £750 stock, at 75 per cent., allowing the broker $\frac{3}{8}$ per cent. for his trouble? *Ans.* £565 „ 6*s.* 3*d.*

7. What will the insurance on £1255 amount to at $7\frac{3}{4}$ per cent.? *Ans.* £97 „ 5*s.* 3*d.*

8. What must I pay my agent for selling goods to the amount of £758 „ 17*s.* 6*d.*, allowing him a commission of $3\frac{1}{2}$ per cent.? *Ans.* £26 „ 11*s.* $2\frac{1}{2}$*d.*

9. What will the insurance on £1387 „ 10*s.* amount to at $12\frac{3}{4}$ per cent.? *Ans.* £176 „ 18*s.* $1\frac{1}{2}$*d.*

10. Bought goods to the amount of £1250 „ 17*s.* 6*d.*, for which I paid commission 2½ per cent., and freight $\frac{1}{8}$ per cent.; what shall I have to pay in addition to the cost of the goods? *Ans.* £32 „ 16*s.* 8½*d.*

11. What will be the insurance of a ship and cargo, value £28757 „ 17*s.* 6*d.*, amount to at the rate of 17⅞ per per cent.? *Ans.* £5140 „ 9*s.* 4¾*d.*

12. What will be the purchase of £2000, bank annuities, at 89¾ per cent.? *Ans.* £1795.

13. Bought £1500 three per cent. stock, at 85⅛ per cent., and allowed the broker $\frac{3}{8}$ per cent.; what does the purchase amount to? *Ans.* £1282 „ 10*s.*

44. Bought an annuity of £50, terminable in 50 years, at 19⅜ years' purchase; what did I pay for it? *Ans.* £968 „ 15*s.*

15. Bought £500 consols at 87⅜, and re-sold them at 89¾; what had I to receive for the difference? *Ans.* £11 „ 17*s.* 6*d.*

16. What must be paid for 30 fifty-pound shares bearing a premium of 7¾ per cent.? *Ans.* £1616 „ 5*s.*

17. What must be paid for 15 fifty-pound shares that have fallen 15 per cent. below par? *Ans.* £637 „ 10*s.*

18. What number of £50 shares, bearing a premium of 15 per cent., must be given in exchange for 23 thirty-pound shares that have fallen 12½ below par. *Ans.* 10½.

19. What will the purchase of £2500 consolidated bank annuities amount to at 85⅞ per cent. allowing $\frac{1}{8}$ per cent. for brokerage? *Ans.* £2150

20. What must be paid for the purchase of an annuity of £750 at 25¾ years' purchase? *Ans.* £19312 „ 10*s.*

DISCOUNT.

1. Discount is an allowance, at the usual rates of interest, for the payment of a sum of money any time before it is due; and in equity ought to be so calculated that neither party may gain or lose by the transaction.

2. The sum of money paid in advance is called the present worth of the sum to be discounted; and the allowance paid for advancing it is called the discount.

3. The usual practice of deducting for the discount, the interest for the time of the sum to be discounted, however agreeable to common usage, is notwithstanding inequitable.

For, if £100 be put to interest for one year at 5 per cent., the interest at the end of that time will be £5, and the amount £105; consequently £100 is the present worth of £105; and £105 − £100 = £5, is the discount of £105 for 1 year at 5 per cent.

Hence it is evident that £5, the discount of £105, is just equal to the interest of its present worth, £100; consequently, by deducting £5 „ 5*s*., which is the interest of the whole sum to be discounted, the one party receives 5*s*. more than the equitable discount, and the other party £99 „ 15*s*., which is 5*s*. less than the just present worth.

But as the discount of the sum to be discounted, as we have shown above, is just equal to the interest of its present worth for the time, if £5 be deducted for the discount, and £100 be advanced as the present worth, neither party will gain or lose, and consequently the transaction will be mutually equal.

For the discount of £5, if put to interest for 1 year, will amount to £5 „ 5*s*., the exact interest which the one party would have received had he kept the £105 till the end of the year; and the present worth, £100, if put to interest for the same time, will amount to £105, the exact sum which the other party would have received had he waited for it till the end of the year.

4. A correct Rule for the calculation of discount may be easily derived from the principle of Proportion; for the

sum of money to be discounted may always be considered as the amount, and the present worth as the principal put to interest for the time at the given rate per cent.

5. And as the principals and also the interests will always vary directly as the amounts, the present worth and also the discount of a sum of money to be discounted, will be found by Proportion in the given ratio of the amounts.

Thus, to find the present worth of £52 „ 10*s*. at 5 per cent. discount, we have the giv. ra. amts. 105 : 52 10 : : 100 : 50, the present worth of £52 „ 10*s*.; and to find the discount, we have the giv. ra. amts., 105 : 52 10 : : 5 : 2 10*s*., the discount of £52 „ 10*s*. Hence, the

RULE.

1. Find the amount of £100 for the given time and at the given rate at which the sum is to be discounted; and to find the present worth, say, as the amount of £100 is to the sum to be discounted, so is the principal, £100, to the present worth.
2. To find the discount, say, as the amount of £100 is to the sum to be discounted, so is the interest of £100 to the discount; or having found the present worth, as before, subtract it from the given sum, and the remainder will be the discount.

Ex.—To find the present worth of £26 „ 5*s*. for 1 year, discounting at 5 per cent.

Here, adding £5 to £100, we get the amount of £100 for 1 year at 5 per cent. = £105; which is the antecedent in the given ratio of amounts, and obtain the present worth, £25, for the 4th term of the proportion.

```
  £
 100
   5
 ———    £  s       £     £
 105 : 26  5 : : 100 : 25 p. w.
```

Ex. 2.—To find the discount of £26 „ 5*s*. for 1 year at 5 per cent.

Here finding, as before, the amount of £100, we have the given ratio of amounts, in which, taking £5, the interest of £100, as the third term, we get the discount, £1 „ 5*s*., for the fourth term of the proportion.

```
  £
 100
   5
 ———    £  s       £  £ s
 105 : 26  5 : : 5 : 1 5 Disc.
```

EXAMPLES FOR PRACTICE.

1. What is the present worth of £350, due 1 year hence, discounting at 5 per cent.? *Ans.* £333 „ 6*s.* 8*d.*

2. What is the discount of £715 „ 17*s.* 6*d.*, due $\frac{3}{4}$ of a year hence, at 5 per cent.? *Ans.* £25 „ 17*s.* 6*d.*

3. What is the present worth of £171 „ 13*s.* 4*d.*, for 9 months, discounting at 4 per cent.? *Ans.* £166 „ 13*s.* 4*d.*

4. What is the discount of £208 „ 15*s.*, for 15 months, at 3½ per cent.? *Ans.* £8 „ 15*s.*

5. What is the present worth of £79 „ 10*s.*, for 1½ year, discounting at 4 per cent.? *Ans.* £75.

6. The discount of a sum of money for 1 year, at 5 per cent., was £15; what was the present worth? *Ans.* £300.

7. What is the present worth of £821, due 9 months hence, discounting at 3½ per cent.? *Ans.* £800.

8. Discounting at 4½ per cent. a bill due at 18 months, I received £500; what was the amount of the bill?
Ans. £533 „ 15*s.*

9. Bought goods for £350, ready money, and sold them for £380 „ 8*s.* 4*d.*, payable in 9 months; what was the gain ready money, discounting at 5 per cent.?
Ans. £16 „ 13*s.* 4*d.*

10. What is the discount of £63 „ 15*s.* for 15 months, at 5 per cent.? *Ans.* £3 „ 15*s.*

11. Discounting at 5 per cent. a bill of £415, I paid £15 for the discount; at what time was the bill due?
Ans. 9 months.

12. What ready money will discharge a debt of £1658 „ 13*s.* 4*d.*, due 11 months hence, discounting at 4 per cent.?
Ans. £1600.

13. For a bill of £533 „ 15*s.*, due at 18 months, I received £500; at what rate per cent. was the bill discounted? *Ans.* 4½ per cent.

14. If for discounting at 3½ per cent. a bill due at 16 months I received £18 „ 13*s.* 4*d.*; what was the amount of the bill? *Ans.* £418 „ 13*s.* 4*d.*

15. If the present worth of a sum of money, due at 18 months, be £500; what was the sum, and what will be the discount at 4 per cent.?

Ans. £530 the sum, and £30 the discount.

BARTER.

Barter is the exchanging of one kind of merchandise for an equivalent quantity of another kind, instead of buying and selling for money; and as the value of both must be equal, the quantity of goods to be received in exchange will be found by proportion in the inverted ratio of the prices.

Hence all the varieties of Barter come within the range of proportion. If the barter be partly for money and partly for goods, we must deduct from the whole value of the goods, that part of it which is paid for in money, and find by proportion what quantity of the other goods will be equal in value to the remainder.

It will only be necessary to give some

EXAMPLES FOR PRACTICE.

1. How many yards of silk velvet, at 12*s*. 6*d*. per yard, can I have in barter for 750 yds. of narrow cloth, at 8*s*. 6*d*. per yard? *Ans.* 510 yds.

2. What quantity of Irish linen, at 3*s*. 9*d*. per yd., can I have in exchange for 150 yds. of muslin, at 2*s*. 6*d*. per yard? *Ans.* 100 yds.

3. What number of English ells of cloth, at 5*s*. 6*d*. per yd., must be given in barter for 350 Flem. ells of silk, at 8*s*. 3*d*. per yd.? *Ans.* 315 E. ells.

4. If in exchange for 510 yards of silk, at 12*s*. 6*d*. per yd., I receive 750 yds. of cloth; what was the price of the cloth per yd.? *Ans.* 8*s*. 6*d*. per yd.

5. How many tierces of wine, at £125 per pipe, can I have in exchange for 5 hhds. of brandy, at £100 per hhd.? *Ans.* 12 tierces.

6. Sold 15 cwt. of coffee, at 3*s*. 4*d*. per lb., for which I received £150 in money, and the remainder in tea, at 6*s*. 6*d*. per lb.; how many lb. of tea did I receive?
Ans. 400 lb.

7. What quantity of hops, at £3 „ 10*s*. per cwt., must be given in barter for 20 cwt. of tobacco, at 3*s*. 6*d*. per lb.?
Ans. 112 cwt.

8. If I receive in barter 100 hhds. of brandy, at 36*s*. per gal., for 120 pipes of port, what was the price of the port per gallon? *Ans*. 15*s*. per gal.

9. How many reams of paper, at 13*s*. 4*d*. per ream, must be given in barter for 80 pieces of Irish linen, at £1 „ 17*s*. 6*d*. per piece? *Ans*. 225 reams.

10. Sold 1500 reams of paper at 12*s*. per ream, for which I am to receive $\frac{1}{4}$ in money and the remainder in equal quantities of wine at 15*s*, and brandy at 30*s*. per gal.; how much money, and how many gallons of wine and brandy shall I receive.?
Ans. £220 money, and 300 gals. of each.

11. How many hhds. of sugar, weighing each 2 cwt. 1 qr. 14 lb., at £2 „ 16*s*. per cwt., can I have in barter for 30 puncheons of rum at £26 „ 12*s*. per puncheon?
Ans. 120 hhds.

12. If for 112 cwt. of hops, at £3 „ 10*s*. per cwt., I receive 20 cwt. of tobacco; what was the price of the tobacco per lb.? *Ans*. 3*s*. 6*d*. per lb.

13. If I give 375 yards of broad cloth at 30*s*. per yard, for silk at 15*s*., and velvet at 20*s*. per yard, and take twice as much silk as velvet; how many yards of each shall I receive? *Ans*. 450 yds. silk, and 225 yds. velvet.

14. Gave 50 bags of pepper, each weighing 32 lb., at 1*s*. 6*d*. per lb., for £50 in money, and the remainder in raisins at 8*d*. per lb.; what quantity of raisins did I receive? *Ans*. 2100 lb.

15. What quantity of Hollands, at 26*s*. per gallon, must be given in barter for 10 hhds. of brandy, at 39*s*. per gal.?
Ans. 15 hhds.

LOSS AND GAIN.

THIS Rule is only an application of the principles of proportion, in order to calculate the loss or gain in buying or selling goods; or so to adjust the price of goods as to allow any discount which the custom of the trade may require, without interfering with the proposed gain.

Of the former it will be sufficient to give only some examples for practice; but as the latter is frequently misunderstood, we shall in a subsequent chapter examine the principles on which it should be established.

EXAMPLES FOR PRACTICE.

1. If I buy 1 cwt. of sugar for £3 „ 14*s*. 8*d*., and retail it at 9*d*. per lb.; what shall I gain?
Ans. 9*s*. 4*d*.

2. If I buy tobacco at £12 „ 10*s*. per cwt., and sell it for 15 guineas per cwt.; what will be the gain per cent.?
Ans. 26 per cent.

3. If I gain 10 per cent. by selling cloth at 7*s*. 4*d*. per yard, what shall I gain per cent. by advancing the price to 10*s*. 6*d*. per yd.?
Ans. 57½ per cent.

4. If I pay £100 for 500 yards of cloth, at what must I sell it per yard to gain £10 „ 18*s*. 9*d*. by the whole lot?
Ans 4*s*. 5¼*d*.

5. Bought 3 hhds. of tobacco, weighing each 2 cwt. 3 qu. 14 lb., at £14 „ 10*s*. per cwt., and sold it at 3*s*. 4*d*. per lb.; what did I gain?
Ans. £35 „ 18*s*. 9*d*.

6. Bought 5 bales of cloth, each containing 15 pieces, each piece 25 yards, for £1312 „ 10*s*.; what do I gain per cent. by selling the cloth at 17*s*. 6*d*. per yard?
Ans. 25 per cent.

7. Bought goods at £25 per cwt.; at what must I sell them per lb. to gain 12 per cent. on the prime cost?
Ans. 5*s*. per lb.

8. If by selling wine at 14*s.* per gallon I gain 12 per cent., what shall I gain per cent. by advancing the price to 17*s.* 6*d.* per gal.? *Ans.* 40 per cent.

9. At what price per gallon must I sell a hhd. of wine, which cost me £40, and of which 2 pints in every 21 gallons was lost by leakage, so as to gain 24½ per cent. on the prime cost? *Ans.* 16*s.* per gal.

10. Bought goods at 5*s.* 10*d.* per lb., which, being damaged, I can sell only at 4*s.* 8*d.* per lb.; what is the loss per cent.? *Ans.* 20 per cent.

11. Bought 150 pieces of cloth at £5 ,, 15*s.* 6*d.* per piece, sold 60 of the pieces at £5, and 70 at £6 per piece; at what rate per piece must I sell the remainder to gain 25 per cent. by the whole? *Ans.* £18 ,, 2*s.* 9¾*d.* per piece.

12. After adding 24 gallons of water to a pipe of wine, which cost £75, I sold it at 15*s.* per gallon; what did I gain or lose per cent.?

Ans. Gained 50 per cent.

13. If after losing 24 gallons from a pipe of wine, which cost £112 ,, 10*s.*, I sell the remainder at 18*s.* 9*d.* per gallon; what do I gain or lose per cent.?

Ans. I lose 15 per cent.

14. If after adding 24 gallons of water to a pipe, and selling the mixture at 17*s.* 6*d.* per gallon, I gain 31¼ per cent.; what was the prime cost of the pipe? *Ans.* £100.

15. Bought 2 tons of goods at 15 guineas per cwt., and paid charges 1 *s.* 6*d.* per cwt.; sold one-half at prime cost, and exported the remainder, on which I paid insurance 18½ per cent.; at what rate per lb. must I charge them to gain 25 per cent. upon the whole. *Ans.* 4*s.* 1½*d.* $\frac{1}{280}$ f.

PART II.

On the adjustment of prices with reference to the allowance of discounts. Where it is the custom for the manufacturer to allow the factor or retailer a discount off the gross or invoice price of goods, and especially when from great competition in the trade the discount is large in proportion to the intended gain, the greatest precaution is requisite.

For want of a right principle of calculation in adjusting

the gross or invoice price of his goods, it has not unfrequently happened that the manufacturer has given away not only the whole of his intended gain, but also a considerable part of his capital.

Thus, intending to allow a discount of 25 per cent. and to reserve a gain of 5 per cent. on goods which cost him £100, he has imagined that by adding to the prime cost, £100, £25 for the discount and £5 for the gain, he has rightly calculated the gross or invoice price at £130.

But the retailer taking off 25 per cent. from the £130, deducts £32 „ 10*s*. for the discount, and pays the manufacturer £97 „ 10*s*. as the net price; thus, deducting not only the whole of the intended gain, but also £2 „ 10*s*. from the prime cost or capital embarked.

The cause of this error is obvious: the manufacturer calculates the discount on the prime cost, and the retailer calculates it on the gross price; and as the gross price is always greater than the prime cost, the retailer will always take off more than the manufacturer puts on.

To find correctly the gross or invoice price of the goods so as to allow the discount and reserve the gain, we have only to calculate the effects of the discount and the gain; and as the causes will always vary directly as the effects, we shall easily find them by proportion, in the given ratio of the effects.

Now, the effect of allowing a discount of 25 per cent. is evidently to diminish £100 to £75; and the effect of gaining 5 per cent. is obviously to increase £100 to £105; hence to find the true gross or invoice price, we have in this example, $\overline{100-25} : \overline{100+5}$, or $75 : 105$, the given ratio of effects; and by proportion, as $75 : 105 :: 100 : 140$£, the true gross or invoice price.

That £140 is the true gross price is evident; for deducting from £140, the discount at 25 per cent., which is just $\frac{1}{4}$ or £35; we have £140 − £35 = £105, the net price paid to the manufacturer, which includes £100, the prime cost, and also £5, his intended gain at 5 per cent.

Or, still more briefly, if after calculating all his expenses, and adding to the amount, the gain he intends to receive, we call this the net price which the manufacturer

is to receive after the discount is deducted; to find the true gross price, he has only to say as £100 – the discount is to £100, so is the net price to the gross price.

Thus, if the goods, including all charges, cost the manufacturer £150, and he intends to gain 15 per cent. on this sum; then £150 the prime cost, + £22 „ 10*s*., the gain, = £172 „ 10*s*., the net price; and to find the gross price from which he may allow a discount of 25 per cent., and secure the net price, he has only to say, as £100 – £25, or 75 : 100 : : 172 „ 10 : 230£, the pure gross or invoice price of the goods.

For if from £230 we deduct the discount at 25 per cent., which amounts to £57 „ 10, we have £230 – £57 „ 10 = £172 „ 10*s*., the whole of the net price, which includes £150, the prime cost, and also £22 „ 10*s*., the intended gain, and is therefore the true invoice price of the goods.

EXAMPLES FOR PRACTICE.

1. The prime cost of goods is £100; at what must they be rated to gain 15 per cent. and allow a discount of 25 per cent.? *Ans*. £153 „ 6*s*. 8*d*.

2. At what price per dozen must goods be sold which cost the manufacturer 24*s*. per doz., that he may allow 20 per cent discount, and gain 25 per cent.?
Ans. £1 „ 17*s*. 6*d*. per doz.

3. If the prime cost of goods be £125; at what must they be sold to allow a discount of 25 per cent. and secure a gain 17½ per cent.? *Ans*. £195 „ 16*s*. 8*d*.

4. A manufacturer makes goods which cost him £100, and wishing to gain 20 per cent. and allow a discount of 25 per cent., sells them for £145; what does he gain or lose? *Ans*. He gains 8¾ per cent.

5. A manufacturer makes goods at 12*s*. 6*d*. per hundred and sells them at 15*s*. per hund., subject to a discount of 30 per cent.; what does he gain or lose?
Ans. He loses 16 per cent.

6. At what should goods be rated which cost the manufacturer 12*s*. 6*d*. per hundred that he may allow 30 per cent. discount, without either loss or gain?
Ans. 17*s*. 10¾*d*. per hund.

7. What discount may be allowed on goods which cost the manufacturer £17 „ 17*s.* 6*d.*, and are sold at £26 „ 16*s.* 3*d.*, to secure to him a gain of 35 per cent.?

Ans. £10 discount.

8. If by selling goods at 25*s.* 6*d.* per lb., and allowing a discount of 17½ per cent., I gain 35 per cent., what was the prime cost? *Ans.* 15*s.* 7*d.*

9. If goods cost the manufacturer 10 guineas per gross, at what must they be sold to gain 35 per cent., and allow a discount of 25 per cent.? *Ans.* 18 guineas.

10. If upon selling goods at 18 guineas per gross, subject to a discount of 25 per cent., there be a gain of 35 per cent., what was the prime cost?

Ans. 10 guineas.

11. A person selling goods which cost him £100 for £150, subject to a discount of 30 per cent., with one year's credit, agrees to allow 5 per cent. for ready money, and accordingly allows the purchaser to deduct 35 per cent. from the price of the goods; how much does he lose by so doing? *Ans.* £2 „ 10*s.*

12. In the manufacture of articles of silver, weighing 1 oz. each; the silver costs 5*s.* 3*d.* per oz., the workmen's wages £1 „ 4*s.* per dozen, and in every dozen 5 dwts. of silver are lost: at what rate per gross must the manufacturer charge them to allow a discount of 35 per cent. and to secure a gain of 30 per cent.?

Ans. £105 „ 19*s.* 6*d.* per gross.

FELLOWSHIP.

Is the name usually given to the rule for dividing the proceeds of any joint concern among the several partners in proportion to their respective interests or shares in the common stock.

This rule is also used to divide a bankrupt's estate among his several creditors, or property under a will among the several legatees, in proportion to their respective claims, when the property may be inadequate to pay them in full, and to other similar purposes.

Fellowship is usually distinguished as "single" or "double." Single fellowship divides the proceeds of the concern among the partners, in the simple ratio of their shares of the joint stock. Double Fellowship divides them in the compound ratio of the shares of stock, and also of the times for which they have respectively been employed in the concern.

From this definition it is evident that fellowship, whether single or double, is nothing more than an application of the principles of proportion to the particular purposes for which it is employed.

We shall therefore give only a detailed example, and leave the student to apply these principles according to his own judgment, as the case may require.

Ex.—A. B. and C., trading with a joint capital of £1200, at the end of one year gain £600; A. advanced £300, B. £400, and C. £500; what is each partner's share of the gain?

Here, adding together the several shares, we have £300 + £400 + £500 = £1200, the whole stock; and in the ratio of the whole stock to each of the shares, we get the shares of

	£.
A.	300
B.	400
C.	500

Wh. Cap., 1200 : 300 : : 600 : 150, A.'s sh.
1200 : 400 : : 600 : 200, B.'s sh.
1200 : 500 : : 600 : 250, C.'s sh.

2nd.

$$\underset{2}{\cancel{12}\cancel{00}} : \underset{1}{\cancel{6}\cancel{00}} : : \begin{cases} 300 : 150, \text{ A.'s sh.} \\ 400 : 200, \text{ B.'s sh.} \\ 500 : 250, \text{ C.'s sh.} \end{cases}$$

the gain, £150, £200, and £250, respectively; or we may

say, as the whole stock is to the whole gain, so is each partner's share of the stock to his share of the gain, as in the 2nd example.

But here it is obvious, that as the whole gain is just one-half of the whole stock; so each partner's share of the gain will be just one-half of his share of the stock.

EXAMPLES FOR PRACTICE.

1. A. advances £750, and B. £1270, and, after trading in partnership for 6 months, gain £505; what is each partner's share? *Ans.* A.'s £187 „ 10*s*., B.'s £317 „ 10*s*.

2. B. and C., after trading together for 12 months, gain £1000; B.'s share of stock is £1250, and C.'s £2500; what is each partner's share of the gain?
Ans. B.'s £333 „ 6*s*. 8*d*., C.'s £666 „ 13*s*. 4*d*.

3. A. and B., trading together with a joint stock of £2020, gain £757 „ 10*s*.; A. received £281 „ 5*s*. for his share of the gain, and B. £476 „ 5*s*.; what was each partner's share of the stock? *Ans.* A.'s £750, B.'s £1270.

4. Divide £1500 into shares, which shall be to each other as 4, 5, and 6. *Ans.* £400, £500, and £600.

5. A bankrupt owes A. £700, B. £850, C. £1450; his whole estate is worth only £1200; how much will each of his creditors receive?
Ans. A. will have £280, B. £340, and C. £580.

6. A person bequeaths £20,000 to be divided among his niece, his nephew, and his son: the nephew is to have three times as much as the niece, and the son four times as much as the nephew; what is the just share of each?
Ans. The niece £1250, the nephew £3750, and the son £15,000.

7. A., B., and C. freight a ship with 510 tuns of wine, of which A. loaded 194 tuns, B. 166, and C. the remainder; the seamen, to save the vessel, were obliged to throw 102 tuns overboard; what part of the loss must each sustain?
Ans. A. $38\frac{4}{5}$, B. $33\frac{1}{5}$, and C. 30 tuns.

8. A ship, worth £2580, of which $\frac{4}{8}$ belonged to A., $\frac{3}{8}$ to B., and the rest to C., being entirely lost, what part of

the loss must each sustain, supposing £1100 to have been insured?

Ans. A. £740, B. £555, C. £185.

9. A person left by will the following legacies:—to A. £1500, to B. £875, to C. £525, and to D. £350, but when the property was sold, it produced only £2437 „ 10*s.*; how much will each receive?

Ans. A. £1125, B. £656 „ 5*s.*, C. £393 „ 15*s.*, D. £262 „ 10*s.*

10. Three persons, forming a joint stock of £45,000, gain by trading £15,000; of this gain, A. receives for his share £7500, B. £5000, and C. £2500; what share of the stock did each advance?

Ans. A. £22500, B. £15000, C. £7500.

11. A., with a capital of £5000, induces B. to join him with £3000, and C. with £2000; after one year the returns were just equal to the expenses, and A. retires from the concern; but in consideration of having led B. and C. into an unprofitable speculation, draws out only half of his capital, leaving the other half to be divided between them in the ratio of their shares; B. and C., after trading another year with their increased fund, gain £1000; what share of the gain will each receive?

Ans. B. £600, C. £400.

12. Two partners, adventuring equal sums of money, gain £3735; A. by agreement was to have 7½ per cent. more than B. of the whole proceeds, in consideration of his taking the whole management of the concern; what was each person's share?

Ans. A.'s £1935, B.'s £1800.

DOUBLE FELLOWSHIP.

This rule divides the whole proceeds of a joint concern among the several partners, not only in proportion to their several shares of stock, but also in proportion to the times for which they have been respectively employed; and consequently, in the compound ratio of the stocks and times, as in the following example.

Ex.—A. and B trading together gain £880; A. advanced £1000 for 8 months, and B. £500 for 6 months; what is each partner's share of the gain?

Here the given ratio of stocks is £1000 : £500, or 2 : 1; and the given ratio of times 8 : 6, or 4 : 3; and compounding these we have $\overline{2 \times 4} : \overline{1 \times 3}$, or 8 : 3, the compound ratio of the stocks and times, in which the whole gain, £880, is to be divided.

Consequently, $\overline{8 + 3}$: 8, or 11 : 8 : : £880 : £640 A.'s share, and $\overline{8 + 3}$: 3, or 11 : 3 : : £880 : £240, B.'s share of the gain, which are to each other in the ratio 8 : 3, or compound ratio of the stocks and times; for 640 : 240 : : 8 : 3. Hence the

RULE.

Multiply each partner's share of the joint stock by the time for which it has been employed in the concern; and adding together these several products, say, as their sum is to each particular product, so is the whole gain or loss to each partner's particular share of it.

Thus, with reference to the example above, multiplying A.'s stock, £1000, by his time,

A. 1000 × 8 = 8000
B. 500 × 6 = 3000

11,$\cancel{0}\cancel{0}\cancel{0}$: 8000 : : 880 : 640 A.'s sh.
and 11,$\cancel{0}\cancel{0}\cancel{0}$: 3000 : : 880 : 240 B.'s sh.

8 months, we get the product 8000; also, multiplying B.'s stock, £500, by his time, 6 months, we get the product 3000, and adding these together we get the sum 11000, which bears to each of the products, the same ratio as the whole gain or loss bears to each of the shares; or the ratios 11 : 8, and 11 : 3.

Examples for Practice.

1. A. and B. make a joint stock, of which A. advanced £1500 for 9 months, and B. £1200 for 6 months; they gain £1150; what is each person's share of that gain?
Ans. A.'s £750, B.'s £400.

2. A. began business with a capital of £1500 on the 1st of January, and on the 15th of March admitted B. with a capital of £1800; they trade together till the 31st of December, and gain £877 „ 10*s.*; what is each person's share? *Ans.* A.'s £450, B.'s £427 „ 10*s.*

3. A., B., and C. rent a pasture for £20 „ 10*s.*; A. sent into it 75 sheep for 3 months, B. 125 for 5 months, and C. 150 for 8 months; what part of the rent must each pay? *Ans.* A. £2 „ 5*s.*, B. £6 5*s.*, C. £12.

4. Three persons enter into partnership for 12 months, at the end of which they gain £9100; A. advances at first £1500, but at the end of 4 months draws out £500; B. at first advances £1200, and at the end of three months £300 more; C. at first advances £900, and at the end of 6 months £600 more; what is each person's share of the gain? *Ans.* A.'s £2800, B.'s £3420, C.'s £2880.

5. Three persons trading with a joint stock, gain £3650; A. advances $\frac{1}{3}$ of the capital for $\frac{1}{4}$ of the time, B. $\frac{1}{4}$ of the capital for $\frac{1}{2}$ of the time, and C. the remainder for the whole time; what is each partner's share of the gain?
Ans. A.'s £486 „ 13*s.* 4*d.*, B.'s £730, and C.'s £2433 „ 6*s.* 8*d.*

6. A. advances towards a joint stock £6000 for 5 months; B. £5000 for 6 months; C. £4000 for 7½ months; and D. £2500 for 12 months; they gain £4760; what is each partner's share? *Ans.* £1190 each.

7. Two persons advancing equal shares of a stock of £1300, gain £650, of which the one receives as his share £250, and the other £400; how long were their shares employed respectively? *Ans.* 5 mo. and 8 mo.

8. A.'s part of a stock of £1300 was employed for 5 months, and B.'s part was employed for 8 months; their shares of gain were equal; what were respectively their shares of stock? *Ans.* A.'s share £800, B.'s share £500.

9. Two persons trading together gain £650; A. advances £800, and B. £500, both employed for the same time; what was each partner's share of the gain?

Ans. A.'s £400, B.'s £250.

10. A prize worth £3825 is to be divided among 9 officers, 12 assistants, and 100 men, in proportion to their pay and the time of their service; the officers, who have £5 per month, have served 9 months; the assistants, who have £2 „ 10*s.* per month, have served 6 months; and the men, who have 30*s.* per month, have served 3 months; what is the share of each individual?

Ans. The officers £225 each, the assistants £75 each, the men £22 „ 10*s.* each.

EQUATION OF PAYMENTS.

Equation of payments is the finding of a time at which several payments due at different times, may be all paid at once; without gain or loss to either party.

The Rule generally given for this purpose, is founded on the supposition, that the interest gained by keeping a sum of money for any time after it becomes due, is a just equivalent for what is lost by paying an equal sum of money, the same time before it becomes due.

But this supposition is erroneous; for by paying a sum of money before it is due, not the interest, but the discount only is lost; and as the interest of a sum of money at 5 per cent. is $\frac{1}{20}$th part, and the discount $\frac{1}{21}$th part only; it is evident, that by this arrangement, the party making the several payments at once, gains more than a just equivalent for his loss.

The true equated time should be such, that the interest of the sums kept after they are due should be exactly equal to the discount of the sums paid before they are due: in which case neither party would gain or lose.

But the finding of such a time, especially when there are several payments, would be so tedious, and the true time, when found, would differ so little from the time found by the general Rule; that notwithstanding the erroneous principle on which it is founded, it will be always preferred as sufficiently accurate, and much more easily applicable to general use.

According to the hypothesis on which the rule is established, it will be obvious, that when two equal sums due at different times are to be paid at once, the whole interval between the times of the several payments must be divided into two equal parts; and that when the sums are unequal, it must be divided into parts which bear to each other the inverted ratio of the sums.

Thus, if £100 due at 6 months, and £100 due at 12 months, are to be both paid at once, the interval between the payments, which is 6 months, will be divided into two equal portions of 3 mo. each, and the equated time will be 9 months; for just as much is gained by keeping £100 3 months after, as is lost by paying £100 3 months before it is due.

But if £200 due at 6 mo., and £100 due at 12 mo., are both to be paid at once, it is obvious, that by keeping £200 for any time after it is due, twice as much will be gained as is lost by paying £100 the same time before it is due; consequently the time for which the £200 is kept after, must be twice as small as the time for which the £100 is paid before it is due.

Thus the interval between the payments, 6 mo., will be divided into the portions 2 mo. and 4 mo.; and the equated time will be 8 months, which is just 2 mo. after the first, and 4 mo. before the last payment is due; for by keeping £200 for 2 mo. after it is due, just as much is gained as is lost by paying £100 4 mo. before it is due.

Hence, it is evident, that when the sums are unequal, the portions into which the interval is divided will also be unequal, and that they will vary inversely as the sums.

RULE.

Multiply each payment by the time at which it becomes due, and divide the sum of the products

by the sum of the payments; the quotient will be the equated time.

Ex.—At what time may £100 due at 6 months, and £100 due at 12 mo. be both paid at once, without gain or loss?

$$\begin{array}{r} 100 \times 6 = 600 \\ 100 \times 12 = 1200 \\ \hline 200 \quad) \; 1800 \\ \hline 9 \text{ mo.} \end{array}$$

Ex. 2.—At what time may £200 due at 6 mo., and £100 due at 12 months, be both paid at once without gain or loss? These need no illustration.

$$\begin{array}{r} 200 \times 6 = 1200 \\ 100 \times 12 = 1200 \\ \hline 300 \quad) \; 2400 \\ \hline 8 \text{ mo.} \end{array}$$

EXAMPLES FOR PRACTICE.

1. £150 due at 6 months, and £150 due at 9 mo. are to be paid at once; what is the equated time?
Ans. 7½ months.

2. £50 is due at 6 mo., £60 at 7 mo., and £80 at 10 mo.; at what time may they be all paid at once?
Ans. 8 months.

3. At what time may £200 due at 3 mo., £300 due at 8 mo., and £500 due at 12mo., be all paid at once?
Ans. 9 months.

4. Of a debt of £1500 due at 15 months, £750 was paid at 6 mo.; at what time should the remainder be paid?
Ans. At 24 months.

5. Of a debt of £1200 due at 12 months, £900 was paid at 9 mo.; at what time should the remainder be paid?
Ans. At 21 months.

6. A debt of £900 due at 6 months, was discharged by paying a part at 3 mo., and the remainder at 9 months; what was the amount of each payment?
Ans. £450 each.

7. A debt of £1200 due at 9 months, was discharged by paying a part in 6 mo., and the remainder at 15 mo.; what was the amount of each payment?
Ans. 1st payment £800, the 2nd £400.

8. A. owed B. £750, to be paid in 15 months, but at 12 mo. paid him £250; at what time was the remainder due? *Ans.* 16½ months.

9. A. owes B. £3000, due at 4 months, £4000 due at 6 mo., and £5000 due at 12 mo.; what will be the equated time to pay the whole? *Ans.* 8 months.

10. Of a debt of £1200, ½ was due at 3 mo., ¼ at 4 mo., ⅛ at 5 mo., and the remainder at 7 mo.; at what time may the whole be paid at once? *Ans.* 4 months.

11. Of a debt of £1200 due at 9 months, £800 was paid before, and the remainder after the time; what was the time of each payment?
Ans. The 1st 4½ months, the 2nd 18 months.

12. Of a debt of £800 due at 15 months, part was paid at 7½ mo., and part at 67½ mo.; what was the amount of each payment? *Ans.* The 1st £700, the 2nd £100.

ALLIGATION.

1. This is simply an application of the principles of proportion, to find either the value of a mixture consisting of several ingredients of different values; or the quantities of the several ingredients which, according to their respective values, must be taken to form a mixture of any required value.

2. The former of these is nothing more than finding the values of the several ingredients at their respective prices, and dividing the sum of these values by the sum of the quantities; in order to find the price of a lb., an oz., or gal., &c. of the mixture, which is called the mean rate.

Thus, to find the value of 1 lb., or the mean rate of a mixture consisting of 12 lb. of tea, at 4*s.* per lb., and 24 lb. of tea at 7*s.* per lb.

Here multiplying the 12 lb. of tea by the price 4*s*., we get their value 48*s*.; and multiplying the 24 lb. by the price 7*s*., we get their value 168*s*. Now adding together the values 48*s*. and 168*s*., we get 216*s*., the value of the whole mixture; and lastly, dividing this value 216*s*. by the sum of the quantities, $12+24=36$, the number of lb. in the whole mixture, we get the quotient 6*s*., which is the price of 1 lb., or the mean rate of the mixture.

$$\begin{array}{rcl} 12 \times 4 & = & 48 \\ 24 \times 7 & = & 168 \\ \hline 36\ ) & & 216\ (6s.\ \text{per lb.} \\ & & 216 \\ \hline \end{array}$$

3. The latter part of the Rule, from which more especially it takes its name, Alligation, from the linking or connecting together the several ingredients which counteract each other's influence, we shall examine separately; of the former we shall give only some

EXAMPLES FOR PRACTICE.

1. A grocer mixes 24 lb. of sugar at 6*d*., 48 lb. at 7*d*., and 96 lb. at 9*d*. per lb.; what is the mean rate of the mixture, or the price per lb.? *Ans*. 8*d*.

2. A grocer mixes 6 lb. of coffee at 1*s*. 3*d*. per lb. with 12 lb. at 1*s*. 6*d*., and 18 lb. at 1*s*. 9*d*. per lb.; what is the price of the mixture per lb.? *Ans*. 1*s*. 7*d*. per lb.

3. If three chests of tea of equal weight, respectively worth 6*s*., 9*s*., and 12*s*. per lb. be mixed together; what will be the value of 1 lb. of the mixture? *Ans*. 9*s*.

4. A vintner mixes 30 gallons of wine at 12*s*., 40 gals. at 18*s*., and 50 gals. at 24*s*. per gal.; what is one gallon of the mixture worth? *Ans*. 19*s*. per gal.

5. A mealman mixes 120 bushels of flour, at 2*s*. 6*d*. per bushel, with 360 bushels at 3*s*. 6*d*., and 240 bushels at 4*s*. 6*d*. per bushel; what is the mixture worth per bushel? *Ans*. 3*s*. 8*d*. per bushel.

6. A goldsmith melts 15 oz. of gold 20 carats fine, with 10 oz. 22 carats fine, and 5 oz. of pure gold which is 24 carats fine; how many carats fine is the mixture? *Ans*. $21\frac{1}{3}$ carats fine.

7. A vintner mixes 16 gallons of wine, at 10*s*. per gal., with 20 gals. at 15*s*., 25 gals. at 18*s*., and 4 gals. of water, at 0 per gal.; what is a gallon of this compound worth?

Ans. 14*s*. per gal.

8. A composition, consisting of 4 oz. 10 dwts. of gold, at £4 per oz.; 16 oz. 15 dwts. of silver, at 5*s*. 6*d*. per oz.; and 24 oz. 15 dwts. of copper, 1*d*. per oz., is manufactured into pencil cases, weighing each one oz.; how much more than the value of the metal does the manufacturer charge by now selling them at 12*s*. each?

Ans. 2*s*. 1$\frac{3}{4}$*d*.

PART II.

To find the quantity of each of the ingredients, which, in proportion to its value, must be taken, to form a mixture of any mean rate required.

1. It is obvious, that if the ingredients which enter into the mixture, are all above or all below the mean rate, no possible combination of them can produce a mixture of the rate required.

Thus, no possible combination of ingredients at 7*s*., 8*s*., and 9*s*. per oz., can produce a mixture worth 6*s*. per oz.; nor of ingredients at 3*s*., 4*s*., and 5*s*. per oz. produce a mixture worth 6*s*. per oz.

Hence the principle of the Rule in every case, requires that some of the ingredients be above, and others below the mean rate of the mixture; and the whole of it consists in so proportioning the quantities of those which are above, and of those which are below the mean rate, that the joint effect of the one to increase, may be counteracted by the joint effect of the others, to decrease the mean rate, and consequently preserve it unaltered.

For this reason, every ingredient of which the price is above, must be linked with one of which the price is below the mean rate; and when the one ingredient is just as much above as the other is below the mean rate, the quantities of both must be equal.

Thus, in mixing wine at 6*s*. with wine at 4*s*. to make a mixture worth 5*s*. per gallon; every gal. of the 6*s*. wine will increase the mean rate by 1*s*., and every gal. of the 4*s*. wine will decrease it by 1*s*.; hence, as every gal. of the one counteracts the effect of every gal. of the other, it

is evident that the quantity of both ingredients must be equal.

Ex.—1 gal at 6*s*., and 1 gal. at 4*s*.=2 gals. worth 10*s*., which is at the rate of 5*s*. per gal. on the mean rate.

But when the one ingredient is more above, than the other is below the mean rate; or generally when their differences from the mean rate are unequal, the quantities of the ingredients will also be unequal; and in this case will always vary inversely as their differences.

Thus, in mixing wine at 7*s*. with wine at 4*s*., to make a mixture worth 5*s*. per gallon, every gal. of the 7*s*. wine will increase the mean rate by 2*s*., while every gal. of the 4*s*. wine decreases it by 1*s*. only; hence, as it will require 2 gals. of the 4*s*. wine to counteract the effect of every gal. of the 7*s*. wine; it is evident that the quantities of the ingredients will vary inversely as their differences.

Ex.—1 gal. at 7*s*. and 2 gals. at 4*s*.=3 gals. worth 15*s*., which is at the rate of 5*s*. per gal. or the mean rate.

For this reason, in writing down the rates of the several ingredients, to find their differences from the mean rate of the mixture, the difference of each ingredient is written, not opposite to that whose difference it is, but opposite to that ingredient of contrary tendency with which it is linked.

The Rule is founded entirely on the obvious principle that the quantities of the several ingredients will vary inversely as their differences; and it is unnecessary to subdivide it into "Cases;" for after having obtained any mixture at the mean rate required, if we wish to alter either the quantity of the whole compound, or of any one particular ingredient, we have only to alter all the other quantities in the same ratio.

RULE.

1. Write down under each other the rates of the several ingredients, for comparison with the mean rate written in a place convenient for the purpose, and connect with a link, the rate of every ingredient which is above, with one which is below the mean rate.
2. Compare the rate of each ingredient with the mean rate, and write its difference opposite to that

rate with which it is linked; and the several differences thus written, will show the quantities of the several ingredients which must be taken to form the mixture.

3. If there be only one ingredient of which the rate is above, or only one of which the rate is below the mean rate, all the others must be linked with this one; and the sum of their several differences, written opposite to it, will be the quantity of that ingredient; and the difference of that ingredient from the mean rate, being written opposite to each of the others, will show the quantity of each of them to be taken to form the mixture.

Ex. 1.—What quantity of tea at 3*s*., 4*s*., 7*s*., and 8*s*. per lb., must be taken to make a mixture worth 6*s*. per lb.?

			Proof.
	3	2 lb. at	3 = 6
6*s*.	4	1 lb. -	4 = 4
	7	2 lb. -	7 = 14
	8	3 lb. -	8 = 24
		8	) 48
			6*s*.

Here, writing down the rates of the several ingredients 3, 4, 7, and 8, under each other, and placing the mean rate 6*s*. in a situation convenient for comparison, we link the 3*s*. which is below, with the 8*s*. which is above, and also the 4 which is below, with the 7*s*. which is above the mean rate. 2nd. Comparing the 3*s*. with the mean rate 6*s*., we write the difference 3 opposite to the 8*s*. with which it is linked; comparing in like manner the 4*s*. with the mean rate 6*s*., we write the difference 2 opposite to the 7*s*.; also comparing 7*s*. with the mean rate 6*s*., we write the difference 1 opposite to the 4*s*.; and comparing 8*s*. with the mean rate 6*s*., we write the difference 2 opposite to the 3*s*. with which it is linked; and these differences, as thus written, show respectively the quantity of the ingredients against which they stand, which must be taken to form the mixture; or that there must be 2 lb. at 3*s*., 1 lb. at 4*s*., 2 lb. at 7*s*., and 3 lb. at 8*s*., to make a mixture of the mean rate, 6*s*. per lb.

That these are the just proportions, will be evident by inspecting the Proof; for the whole mixture is 8 lb., which at their respective prices are together worth 48*s*., which is at the rate of 6*s*. per lb., or at the mean rate required.

If it had been required that this mixture should contain 240 lb.; having obtained the proportions as above, forming a mixture of 8 lb., we have only to increase the quantity of each ingredient in the ratio 8 : 240 = 1 : 30; or if it had been required, that 21 lb. of the 8*s.* tea should be contained in the mixture, we have only to increase the quantity of all the other ingredients in the ratio 3 : 21 = 1 : 7, and we shall obtain the mixture required.

Ex. 2.—What quantities of tea, at 3*s.*, 4*s.*, 5*s.*, and 8*s.* per lb., will form a mixture worth 7*s.* per lb.

					Proof.
	3	. . .	1	at	3 = 3
7*s.*	4	. . .	1	–	4 = 4
	5	. . .	1	–	5 = 5
	8	4 + 3 + 2 =	9	–	8 = 72
			12	)	84
					7*s.*

Here, proceeding as before, we find the only ingredient of which the rate is above the mean rate, to be the tea at 8*s.*; consequently all the other rates are linked with it, and their differences from the mean rate, 4, 3, and 2, are all written opposite to it; their sum 4 + 3 + 2 = 9, shows the quantity of that ingredient to be used in the mixture; and the difference of that ingredient from the mean rate 7 = 1, is written opposite to each of the ingredients which are linked with it, and shows the quantity of each which must be taken to form the mixture required; thus 9 lb. at 8*s.*, and 1 lb. each at 3*s.*, 4*s.*, and 5*s.*, together constituting a mixture of 12 lb. worth 84*s.*, or at the mean rate 7*s.* per lb., as may be seen by inspecting the Proof.

EXAMPLES FOR PRACTICE.

1. A vintner would mix wines at 14*s.* and 16*s.* per gallon, so as to sell the compound at 15*s.* per gallon; what quantity of each must he take?

Ans. Equal quantities of each.

2. A distiller mixes spirits at 7*s.*, 8*s.*, 10*s.*, and 12*s.* per gallon, and sells the mixture at 9*s.* per gal.; how many gals. of each wine did he use?

Ans. 3 g. at 7*s.*, 1 g. at 8*s.*, 1 g. at 10*s.*, and 2 gs. at 12*s.*

3. A mealman mixes flour at 3*s.* 4*d.*, 3*s.* 8*d.*, 4*s.*, and 4*s.* 3*d.* per bushel; what quantity of each did he take to make the mixture worth 3*s.* 10*d.* per bushel?

Ans. 6 at 3*s.* 4*d.*, 2 at 3*s.* 8*d.*, 2 at 4*s.*, and 6 at 4*s.* 3*d.*

4. A grocer mixes teas at 4*s*., 4*s*. 6*d*., 5*s*., 5*s*. 6*d*. and 6*s*. per lb., to sell the mixture at 4*s*. 9*d*. per lb.; how much of each did he take?

Ans. 24 lb. at 4*s*., 3 lb. at 4*s*. 6*d*., 3 lb. at 5*s*., and 9 lb. each at 5*s*. 6*d*. and 6*s*.

5. How many ounces of gold, 17, 18, 22, and 24 carats fine, will make a mixture of 20 carats fine?

Ans. 4 at 17, 2 at 18, 2 at 22, and 3 oz. at 24 carats.

6. A grocer would mix teas at 4*s*., 6*s*., 8*s*., and 10*s*., so as to form a compound of 56 lb., worth 7*s*. per lb.; what quantity of each will be requisite?

Ans. 21 lb. each, at 4*s*. and 10*s*., and 7 lb. each, at 6*s*. and 8*s*.

7. What quantity of flour at 2*s*. 9*d*., 3*s*. 6*d*., and 4*s*. 3*d*. must be taken to form a mixture worth 3*s*. 9*d*. per bushel?

Ans. 6 bu. each at 2*s*. 9*d*. and 3*s*. 6*d*., and 15 bu. at 4*s*. 3*d*.

8. How many lb. of coffee at 2*s*. 6*d*., 3*s*. 4*d*., and 3*s*. 8*d*., must be mixed with 45 lb. at 4*s*. 6*d*. per lb. to make a mixture worth 4*s*. per lb.?

Ans. 9 lb. of each.

9. How much water at 0, and wine at 16*s*., 18*s*., and 20*s*. per gallon, will form a mixture worth 17*s*. per gal.?

Ans. 3 g. water, 1 ga. ea. of wine at 16*s*. and 18*s*., and 17 gal. at 20*s*.

10. How many gallons of wine at 12*s*., 15*s*., 17*s*., and 24*s*. must be mixed together, that by selling the compound at 25*s*. per gal., the merchant may gain 25 per cent.?

Ans. 14 gal. at 24*s*., and 4 gal. of each of the others.

11. A merchant mixing brandies at 25*s*., 35*s*., and 45*s*. per gal., wishes to make a mixture of 210 gallons, worth 2 guineas per gal.; what quantity of each must he take?

Ans. 168 gal. at 45*s*., and 21 gal. of each of the others.

12. A merchant mixing wines at 12*s*., 15*s*., 17*s*., and 24*s*., sells the mixture at 17*s*. 6*d*. per gallon; how many gallons of each did he use?

Ans. 8½ gal. at 24*s*., and 6½ gal. of each of the others.

13. What quantity of spirits at 17*s*., 19*s*., and 21*s*. per

gallon, and of water at 0, must be taken to form a mixture by the sale of which at 18*s*. per gal. the merchant may gain 20 per cent.?

Ans. 15 ga. of each spirit, and 12 ga. water.

14. How many ounces of gold at £4 per oz., silver at 5*s*. 6*d*., and copper at 1*d*. per oz., will form a metallic compound worth 17*s*. 6*d*. per oz.?

Ans. 353 oz. gold, 750 oz. silver, and 750 oz. copper.

EXCHANGES.

This Rule is merely an application of the principles of proportion, to find what quantity of the money of one country, will be equal in value to a given quantity of the money of any other country, according to the usual rate of exchange.

Many circumstances connected with this subject, do not properly come within the province of an elementary treatise on arithmetic, of which the design is rather to teach a right use of them when known, than to anticipate the various usages which can be learned only from actual business.

It will therefore be sufficient to notice only the following, which enter more materially into the calculation of questions in exchanges.

1. Agio is an allowance of so much per cent. for the difference between the value of the money kept in the banks, and that which is in circulation; the former, called banco, being more valuable than the latter, which is called currency.

2. The exchange is always supposed to be made in banco, to which, according to the agio, every sum of currency must be reduced, before the exchange is made. Sterling money is always considered to be equal to banco.

3. Usance is a certain time allowed for the payment of bills of exchange; and is generally 15 days, or a month, or two months from their date, according to the distance of the place; and has the same effect on the sum as the discounting of it for the time.

4. The Par of exchange is such a mutual adjustment of the relative values of the moneys of two countries, as may make the rate of exchange, which is subject to fluctuate, equally advantageous to both.

5. The course of exchange is that fluctuation from par, which, according to circumstances, makes the rate of exchange advantageous to the one country, and disadvantageous to the other.

In pursuing this subject, it will be requisite to subjoin tables of the relative values of the different coins; and, after detailing an example, it will be sufficient to leave the student to proceed according to his own discretion.

ENGLAND, WITH HOLLAND, FLANDERS, AND GERMANY.

Accounts are kept in these countries in guilders, stivers, grotes, and pennings; or in pounds, schillings, and pence Flemish. The course of exchange varied from 33*s*. 6*d*. to 36*s*. 6*d*. Flemish, per £. sterling; and the agio from 3 to 6 per cent.

TABLE.

8 Pennings	=	1 grote or penny Flemish.
2 Grotes	=	1 Stiver.
6 Stivers	=	1 Schilling.
20 Stivers	=	1 Guilder or Florin.
$2\frac{1}{2}$ Guilders	=	1 Rix-Dollar.
6 Guilders	=	1 Pound Flemish.

Ex.—How many guilders currency, are equal in value to £150 sterling; exchange at 35*s*. 6*d*. Flemish per £. sterling, and agio 5 per cent.?

Here we find by Practice, that the value of £150 sterling, at 35*s*. 6*d*. per £., is = £266 „ 5*s*. Flem. banco; and as the agio is 5 per cent., we add $\frac{1}{20}$ of this sum to the banco, and find the currency = £279 „ 11*s*. 3*d*. Flem., which we reduce into guilders by multiplying by 6, and obtain the answer, 1677 guilders, 7 stivers, and $\frac{6}{12}$ = 1 grote.

	£	*s*.		
	150	0		
10*s*. $\frac{1}{2}$ =	75	0		
5*s*. $\frac{1}{4}$ =	37	10		
6*d*. $\frac{1}{10}$ =	3	15		
	£266	5		Fl. Ban.
Agio = $\frac{1}{20}$ =	13	6	3	
	£279	11	3	Fl. Cur.
			6	
Guild.	1677	7	6	

EXAMPLES FOR PRACTICE.

1. In 1677 guilders $7\frac{1}{2}$ stivers currency, how many £. sterling, exchange 35*s*. 6*d*. Flem. per £. sterling, and agio 5 per cent.? *Ans*. £150 sterling.

2. In £196 „ 13*s*. 4*d*. sterling, how many guilders banco, exchange at 34*s*. 4*d*. Flem. per £. sterling?
Ans. Guilders 2025 „ 13 st. $5\frac{1}{3}$ pen.

3. How much sterling is equivalent to 2345 guilders agio, 15 stivers banco, exchange 33*s*. 9*d*. Flem. per £. sterling? *Ans*. £231 „ 13*s*. $6\frac{3}{4}$*d*. $\frac{1}{4}$.

4. How many rix-dollars currency are equivalent to £150 „ 10*s*. 6*d*. sterling, exchange 36*s*. 3*d*. Flem. per £. sterling, and agio $5\frac{1}{2}$ per cent.?
Ans. 690 r. d. 1 guild. 19 st. 1 gr. 5 pen.

5. If the exchange be 33*s*. 6*d*. Flem. per £. sterling, at $1\frac{1}{2}$ month's usance, what should it be at sight, allowing interest at 4 per cent.? *Ans*. 33*s*. 4*d*.

6. In £4567 „ 17*s*. 6*d*. Flem. currency, how many £. sterling, exchange 35*s*. 9*d*. Flem. per £. sterling, and agio $4\frac{7}{8}$ per cent.? *Ans*. £2436 „ 13*s*. 4*d*.

HAMBURG AND ALTONA.

They keep accounts here in rix-dollars, marks, schillings and fennings, and exchange with London by the pound Flemish, which varies from 33 to 36 schillings, Flem. per £. sterling; agio for currency, from 18 to 20, and for light money from 30 to 35 per cent.

TABLE.

6 Fennings	=	1 Grote Flemish.
12 Fennings	=	1 Schilling.
16 Schillings	=	1 Mark.
2 Marks	=	1 Hamb. dollar of exchange.
3 Marks	=	1 Rix-Dollar.
$7\frac{1}{2}$ Marks	=	1 Pound Flemish.

EXAMPLES FOR PRACTICE.

1. Reduce 357 marks 12 shillings banco, into £.'s sterling, exchange 33*s*. 6*d*. Flem. per £. sterling?
Ans. £28 „ 9*s*. 6$\frac{1}{2}$*d*. $\frac{84}{67}$.

2. How many marks banco are equivalent to £175 „ 12*s*. 6*d*. sterling, exchange at 33*s*. 4*d*. Flemish per £. sterling? *Ans*. 2195 marks 5 shil.

3. How much sterling will purchase 3500 marks currency, exchange 35*s*. Flem. per £. sterling, and agio 20 per cent.? *Ans*. £222 „ 4*s*. 5$\frac{1}{2}$*d*.

4. How many rix-dollars currency are equivalent to £250 sterling, exchange 34*s*. 6*d*. Flem. per £. sterling, and agio 18 per cent.? *Ans*. 1272 rix-dol. 9 shil.

5. How many dollars of exchange currency are equivalent to £125 sterling, agio 20 per cent., and exchange 36*s*. Flem. per £. sterling?
Ans. 1012 dol. ex. 8 shil.

6. In 1720 Hamburg rix-dollars, how many £. Flem.?
Ans. £688 Flemish.

SWEDEN.

Accounts at Stockholm are kept in copper and silver dollars, and the exchange varies from 46 to 50 copper dollars per £. sterling.

TABLE.

2 Fennings	=	1 Runstychen.
6 Runstychens	=	1 Skilling.
1$\frac{1}{3}$ Skil., or 8 Runstych.	=	1 Mark.
4 Marks	=	1 copper Dollar.
3 copper Dols., or 16 skils.	=	1 silver Dollar.
3 silver Dollars	=	1 Rix-dollar.

EXAMPLES FOR PRACTICE.

1. In 5922 cop. dol. 3 marks 4 runstych., how much sterling, exchange 49 cop. dollars per £. sterling?
Ans. £120 „ 17*s*. 6*d*.

2. In £175 „ 15*s.* 6*d.* sterling, how many silver dollars, exchange 48½ cop. dols. per £. sterling?

Ans. 2841 sil. dol. 2 cop. dol. 2 runstych.

3. In £375 „ 17*s.* 6*d.* sterling, how many rix-dollars, exchange 50 cop. dollars per £. sterling?

Ans. 2088 rix-dol. 1 cop. dol. 2 marks 7 runstych. 1 fen. $\frac{94}{100}$.

4. How much sterling is equivalent to 1234 silver dollars, exchange 49½ cop. dols. per £. sterling?

Ans. £74 „ 15*s.* 9*d.*

5. How many rix-dollars are equivalent to the payment of a bill of £500 sterling, 2 months' usance, exchange at 48 cop. dols. per £. sterling?

Ans. 2644 rix-dol. 1 sil. dol. 1 cop. dol.

6. What quantity of sterling must I remit to Stockholm to pay a bill of 1022 silver dollars, exchange 47½ cop. dol. per £. sterling? *Ans.* £64 „ 10*s.* 11¼*d.* $\frac{9}{19}$.

RUSSIA.

They keep accounts at Petersburg in Rubles and Copecks, and exchange with London direct at from 4*s.* to 5*s.* sterling per ruble, or by way of Amsterdam at from 48 to 50 stivers per ruble.

TABLE.

10 Copecks	=	1 Grieven.
50 Copecks or 5 Grievens	=	1 Politin.
100 Copecks or 2 Politins	=	1 Ruble.
2 Rubles	=	1 Ducat.

EXAMPLES FOR PRACTICE.

1. Reduce 1750 rubles 75 copecks into sterling, exchange at 4*s.* 9*d.* sterling per ruble?

Ans. £415 „ 16*s.* 0¾*d.*

2. How many ducats, &c., are equivalent to £218 „ 16*s.* 4¾*d.* sterling, exchange at 4*s.* 9*d.* sterling per ruble?

Ans. 460 duc. 1 rub. 35 copecks

3. How much sterling is equivalent to 1842 rubles 70 copecks, exchange at 4*s*.10½*d*. sterling per ruble?
Ans. £449 „ 3*s*. 1¾*d*. $\frac{8}{10}$.

4. In £137 „ 15*s*. sterling, how many rubles, &c., exchange 48 stivers of Amsterdam per ruble, and 33*s*. 4*d*. Flem. per £. sterling? *Ans*. 573 rub. 95 copecks.

5. How much sterling is equivalent to 2408 rubles, exchange 50 stivers per ruble, and 35*s*. 10*d*. Flem. per £. sterling? *Ans*. £560.

6. A merchant at London remits to his correspondent at Petersburg £375 „ 17*s*. 6*d*. by way of Amsterdam, exchange 34*s*. 8*d*. Flem. per £. sterling, and thence to Petersburg, at 49 stivers per ruble; with what sum should his account be credited at Petersburg?
Ans. 1595 rubles, 55 copecks.

PRUSSIA.

They keep accounts at Dantzic in Florins, Groschens, and Fennings, and exchange with London by way of Amsterdam, at from 240 to 290 Groschens per £. Flemish.

Table.

18 Fennings	=	1 Groschen.
30 Groschens	=	1 Florin.
3 Florins	=	1 Rix-dollar.
2 Rix-dollars	=	1 Gold Ducat.

EXAMPLES FOR PRACTICE.

1. In £175 sterling, how many Prussian florins, 33*s*. 4*d*. Flem. per £. sterling, and 240 groschen per £. Flemish?
Ans. 2333 flor. 10 gros.

2. How many rix-dollars in £478 „ 14*s*. 9*d*. sterling, exchange at 255 groschen per £. Flemish, and 33*s*. 6*d*. Flemish per £. sterling? *Ans*. 2272 rix-dollars.

3. In 560 gold ducats, how much sterling, exchange 280 groschen per £. Flemish, and 34*s*. 2*d*. Flemish per £. sterling? *Ans*. £210 „ 14*s*. 7½*d*.

4. How many Prussian florins are equivalent to £546 „ 17*s*. 6*d*. sterling, exchange 275 groschen per £. Flemish, and 35*s*. 10*d*. Flemish per £. sterling?

Ans. 6981 flor. 19 gros. 15 fen.

5. In 1235 gold ducats 1 rix-dol. 2 flor., how much sterling, exchange 285 groschen per £. Flemish, 36*s*. 6*d*. Flemish per £. sterling? *Ans*. £427 „ 13*s*. 8½*d*.

6. A merchant remits a bill of £315 sterling at 1 months' usance to Dantzic, to pay for the purchase of goods; with how many gold ducats will his account be credited, exchange 270 groschen per £. Flemish, and 35*s*. Flem. per £. sterling? *Ans*. 823 gold duc. 1 Rx.-dol. 17 groschen.

VENICE.

They keep accounts here in Ducats, Lire, Soldi, and Denari, and exchange with London at from 52*d*. to 54*d*. sterling per ducat, and from 45*d*. to 54*d*. sterling per lira or piastre of Leghorn, agio from 20 to 30 per cent.

TABLE.

12 Denari	=	1 Soldi.
20 Soldi	=	1 Lira or Piastre.
6¼ Lire	=	1 Ducat current.

EXAMPLES FOR PRACTICE.

1. What is the value in sterling of 3456 piastres of Leghorn, 15 soldi, and 9 denari, exchange at 48*d*. sterling per piastre? *Ans*. £691 „ 7*s*. 1¾*d*.

2. How many piastres of Leghorn are equivalent to £178 „ 17*s*. 6*d*. sterling, exchange at 50*d*. sterling per piastre? *Ans*. 858 piast. 12 soldi.

3. How many ducats current are equal in value to £350 „ 15*s*. 8*d*. sterling, exchange 52*d*. sterling per ducat, and agio 20 per cent.? *Ans*. 1942 duc. 16 soldi.

4. In 1582 ducats current, how much sterling, exchange at 54*d*. sterling per ducat, and agio 22½ per cent.?

Ans. £290 „ 11*s*. 5*d*.

5. How many piastres of Leghorn are equivalent to £120 sterling, exchange 48*d.* sterling per piastre, and agio 20 per cent.? *Ans.* 500 piastres.

6. How much sterling can I have for 6500 ducats current, exchange $52\frac{1}{2}$ sterling per ducat, and agio 30 per cent.? *Ans.* £1093 ,, 15*s.*

FRANCE.

They formerly kept their accounts at Paris in Livres, Sous, and Deniers, which is still the practice in some parts of the country; but at present accounts are generally kept in Francs, Decimes, and Centimes. The exchange with London varies from 24 to 25 francs per £. sterling. The new franc bears to the old livre the ratio of 81 to 80; hence 80 francs are equivalent to 81 livres.

TABLE.

12 Deniers	=	1 Sou.
20 Sous	=	1 Livre.
3 Livres	=	1 Ecu.
and		
10 Centimes	=	1 Decime.
10 Decimes or 100 Cent.	=	1 Franc.

EXAMPLES FOR PRACTICE.

1. Reduce 3831 livres, 17 sous, 9 deniers into £. sterling, exchange at $31\frac{1}{4}d.$ sterling per écu? *Ans.* £162 ,, 12*s.* $10\frac{3}{4}d.$

2. How many francs are equivalent to £125 ,, 10*s.* sterling, exchange at $23\frac{1}{2}$ francs per £. sterling? *Ans.* 2949 francs, 25 centimes.

3. How many £. sterling are equivalent to 1785 francs, 6 decimes, exchange at 24 francs per £. sterling? *Ans.* £74 ,, 8*s*.

4. Reduce 3456 francs into livres, sous, and deniers. *Ans.* 3499 livres, 4 sous.

5. In 1254 francs, how many £. sterling, exchange at $31\frac{1}{4}d.$ pence sterling per écu? *Ans.* £55 ,, 2*s.* $1\frac{3}{4}d.$

6. Reduce $3683\frac{5}{8}$ francs into £. sterling, exchange at $23\frac{1}{2}$ francs per £. sterling? *Ans.* £156 ,, 15*s*.

SPAIN.

They keep accounts at Madrid in Piastres, Reals, and Maravedis, and exchange with London at from 38 to 42 pence sterling per Piastre; the money of Spain is also distinguished as Vellon, which is the Current; and as Plate, which is the Money of Exchange; Vellon bears to Plate the ratio of 17 to 32; hence 32 Reals Vellon are equivalent to 17 Reals Plate.

TABLE.

34 Maravedis	=	1 Real.
8 Reals	=	1 Piastre.
4 Piastres	=	1 Pistole.
375 Maravedis	=	1 Ducat.

EXAMPLES FOR PRACTICE.

1. How many £. sterling are equivalent to 1745 reals plate, exchange at $38\frac{3}{4}$*d.* sterling per piastre?
Ans. £35 „ 4*s.* $4\frac{1}{4}$*d.*

2. Reduce 4352 reals vellon into £. sterling, exchange at 42*d.* sterling per piastre. *Ans.* £50 „ 11*s.* 6*d.*

3. How many piastres are equivalent to £157 „ 13*s.* 6*d.* sterling, exchange at 42*d.* sterling per piastre?
Ans. 901 piastres.

4. In 8704 reals vellon, how many reals plate?
Ans. 4624 reals plate.

5. How many pistoles, &c., are equivalent to £215 „ 15*s.* 6*d.* sterling, exchange 40*d.* sterling per piastre?
Ans. 323 pistoles, 2 piastres, 5 reals, 6 maravedis $\frac{8}{10}$.

6. How many ducats are equivalent to £129 „ 13*s.* 9*d.* sterling, exchange at $41\frac{1}{2}$*d.* sterling per piastre?
Ans. 544 ducats.

7. How much sterling must be remitted to pay a debt of 8704 reals vellon, exchange at 42*d.* sterling per piastre?
Ans. £101 „ 3*s.*

PORTUGAL.

They keep their accounts at Lisbon in Milrees, Crusadoes and Rees, and exchange with London at from 60*d.* to 67*d.* sterling per Milree.

TABLE.

400 Rees	=	1 Crusado.
1000 Rees, or $2\frac{1}{2}$ Crusadoes	=	1 Milree.

EXAMPLES FOR PRACTICE.

1. How many milrees are equivalent to £126 „ 13*s.* 4*d.* sterling, exchange at 64 pence sterling per milree?
Ans. 475 milrees.

2. How much sterling must I remit to pay a debt of 525 crusadoes, 375 rees; exchange at 66*d.* sterling per milree? *Ans.* £57 „ 17*s.* $0\frac{3}{4}$*d.*

3. In £253 „ 6*s.* 8*d.* sterling, how many crusadoes; exchange at 5*s.* 4*d.* sterling per milree? *Ans.* 2375 crusadoes.

4. How much sterling must I remit to Lisbon to discharge a debt of 750 milrees; exchange at 65*d.* sterling per milree? *Ans.* £203 „ 2*s.* 6*d.*

5. How many milrees are there in 12350 crusadoes, 175 rees? *Ans.* 4940 milrees, 175 rees.

6. If I remit to Lisbon £406 „ 5*s.* to discharge a debt of 1500 milrees; what was the rate of exchange?
Ans. 65*d.* sterling per milree.

THE UNITED STATES OF AMERICA.

They keep accounts here in Dollars, Dimes, and Cents, and exchange with London at the rate of 4*s.* 6*d.* sterling per dollar; they also keep accounts in Pounds, Shillings, and Pence currency, the value of which varies greatly in different parts of the country, fluctuating from 4*s.* 8*d.* to 8*s.* currency per dollar of exchange worth 4*s.* 6*d.* sterling, consequently subject to an agio varying from 4 to 77 per cent.

TABLE.

10 Cents	=	1 Dime.
10 Dimes	=	1 Dollar.

EXAMPLES FOR PRACTICE.

1. Reduce 750 dollars, 75 cents into sterling, exchange 4*s.* 6*d.* sterling per dollar. *Ans.* £168 „ 18*s.* 4½*d.*

2. How many dollars currency are equivalent to £120 sterling, exchange at 7*s.* 6*d.* currency per dollar? *Ans.* 320 dollars.

3. Reduce 2750 dollars, 25 cents currency into £. sterling, exchange 4*s.* 6*d.* sterling per dollar, and agio 65 per cent. *Ans.* £375 „ 0*s.* 8*d.*

4. How many dollars current, agio 57½ per cent., are equivalent to £153 sterling, exchange at 4*s.* 6*d.* sterling per dollar? *Ans.* 1071 dollars.

5. With how much sterling can I discharge a debt of 1750 dollars current, agio 75 per cent.? *Ans.* £225.

6. If I remit to Pennsylvania £175 „ 17*s.* 6*d.* sterling, with how many dollars current, agio 66⅔ per cent.? *Ans.* 1302 dollars, 77 centimes.

IRELAND AND THE WEST INDIES.

Accounts are kept here in Pounds, Shillings, and Pence, as in England; but in Ireland, the currency being less valuable than sterling, is subject to an agio varying from 8 to 12 per cent.; and the Paper Currency of the Colonies is also subject to a discount varying from 25 to 50 per cent.

EXAMPLES FOR PRACTICE.

1. Dublin sends goods to London worth £125 „ 12*s.* 6*d.* Irish currency; with how much sterling must she be credited, agio 8⅓ per cent.? *Ans.* £ 115 „ 19*s.* 2¾*d.*

2. London consigns to Virginia goods to the amount of £350 sterling; with how much currency should she be credited, agio 35 per cent.? *Ans.* £472 „ 10*s.*

3. Jamaica remits to London £375 „ 17*s.* 6*d.* currency, agio 27½ per cent.; with how much sterling should she be credited? *Ans.* £294 „ 16*s.* 0¾*d.*

4. How much Irish currency will be equivalent to £500 sterling, agio 11½ per cent.? *Ans.* £557 „ 10*s.* currency.

5. London sends goods to Barbadoes, which are sold for £785 „ 15*s.* currency; how much sterling must the factor remit, agio 31½ per cent., to settle the account?

Ans. £597 „ 10*s.* 6¾*d.*

6. If £750 sterling be equivalent to £836 „ 5*s.* currency, what is the agio? *Ans.* 11½ per cent.

ARBITRATION OF EXCHANGES.

Arbitration of exchanges is the method of determining, from the known courses of exchange between several countries, the proportionate rate of exchange between any two of them; and by comparing the rate thus found with the actual course of exchange, we may determine, according as it may be greater or less, whether a direct or circuitous mode of remittance will be the more advantageous.

Ex.—London is indebted to Paris 3000 écus, exchange 32*d.* sterling per écu; the course of exchange between Paris and Amsterdam is 54*d.* Flem. per écu; and between Amsterdam and London, 36*s.* Flem. per £. sterling; will it be more advantageous for London to remit directly to Paris or circuitously by Amsterdam?

Here, changing the Flemish money into sterling, we find 54*d.* Flem. = to 30*d.* sterling; or the proportionate rate of exchange by Amsterdam to be 30*d.* sterling per écu; and comparing this with the actual rate of exchange, 32*d.* sterling per écu, we find that by remitting circuitously by way of Amsterdam we gain 2*d.* on every écu, which, on 3000 écus, amounts to 6000*d.* = £25 sterling by the circuitous remittance.

$$\begin{array}{ccccccc} s & & s & & d & & d \\ \cancel{36} & : & 20 & :: & \cancel{54} & : & 30 \\ 2 & & 3 & & 3 & & \\ & & \overline{60} & & & & \\ & & \overline{30d.} & \text{ster. per écu.} & & & \end{array}$$

It is sometimes customary to divide this rule into Simple and Compound Arbitration, the former limited to three, and the latter embracing a greater number of places.

But this distinction is altogether unimportant; for whatever may be the number of places, there will be always as

many several ratios as there are different courses of exchange, all of which it will be convenient to reduce and compound into one single ratio, which will express the proportional or arbitrated rate of exchange between the first place and the last.

In arranging these several ratios or courses of exchange, it will be necessary to begin with the course of exchange between the given place and that to which the first remittance is made, and preserving that order from the first to the last place, to see that the consequent of the first ratio be of the same kind as the antecedent of the second; the consequent of the second of the same name as the antecedent of the third, and so on. (See RATIO Art. 14.)

Ex. 2.—London is indebted to Petersburg 500 rubles, exchange at 40*d*. sterling per ruble; but thinking it more advantageous, London remits to Paris at 24 Francs per £. sterling; thence to Portugal, at 500 Rees for 3 Francs; thence to Amsterdam at 20 Stivers per Crusado, and thence to Petersburg at 25 Stivers per Ruble. What does London gain or lose by this circuitous mode of remittance?

1 £ Sterling	=	24 Francs.
3 Francs	=	500 Rees.
400 Rees	=	20 Stivers.
25 Stivers	=	1 Ruble.
1 £ Sterling	=	8 Rubles.

Here, beginning with the course of exchange between London and Paris, we have the ratio of a £. sterling to a Franc = 24 : 1; but as the number of coins will be in the inverted ratio of their values, we have 1 £. sterling = 24 francs; 3 francs = 500 rees, 400 rees = 20 stivers, and lastly 25 stivers = 1 ruble. 2nd. Reducing and compounding these several ratios, we get the compound ratio £1 sterling = 8 rubles, which is the proportional or arbitrated rate of exchange between London and Petersburg = 30*d*. sterling per ruble.

Now, comparing this with the actual rate of exchange, 40*d*. sterling per ruble, it is evident that London, by the circuitous mode of remittance, gains 10*d*. sterling on every ruble, which, on 500 rubles, = 5000*d*. = £20 ,, 16*s*. 8*d*.

EXAMPLES FOR PRACTICE.

1. The course of exchange between London and Amsterdam is 33*s*. 6*d*. Flemish per £. sterling; and between London and Lisbon 50*d*. sterling per milree; what is the

arbitrated rate of exchange between Amsterdam and Lisbon? *Ans.* $83\frac{3}{4}d.$ Flem. per milree.

2. The course of exchange between Petersburg and Amsterdam is 25 stivers per ruble; between Amsterdam and Portugal 20 stivers per crusado, between Portugal and Paris 400 rees per 3 francs; and between Paris and London 25 francs per £. sterling; what is the arbitrated rate of exchange between Petersburg and London?

Ans. $36d.$ sterling per ruble.

3. Taking all the data as given in the preceding question, will Petersburg gain or lose by the circuitous mode of remittance; and how much per cent., supposing the actual course of exchange between Petersburg and London to be $40\frac{1}{2}d.$ sterling per ruble?

Ans. Petersburg will gain $12\frac{1}{2}$ per cent.

4. The course of exchange between London and Amsterdam is $35s.$ $10d.$ Flem. per £. sterling; between Amsterdam and Lisbon, $43d.$ Flem. per crusado; between Lisbon and Paris, 500 rees for 3 francs; and between Paris and Madrid, 14 francs per pistole or doubloon of plate; what is the arbitrated rate of exchange between London and Madrid?

Ans. $140d.$ sterl. per pistole, or $35d.$ per piastre.

5. What will London gain or lose by remitting circuitously, as in the preceding question, for the payment of a debt due to Madrid of 500 pistoles, the actual rate of exchange between London and Madrid being $168d.$ sterling per pistole. *Ans.* £58 „ $6s.$ $8d.$

6. The course of exchange between Paris and London is 24 francs per £. sterling; between Paris and Venice, 512 centimes per piastre; between Venice and Sweden, 102 skillings per piastre; between Sweden and Lisbon, 51 skillings per crusado; and between Lisbon and London, $64d.$ sterling per milree; which is the more advantageous; the direct or circuitous mode of remittance?

Ans. They are equal.

POSITION.

1. Position is a rule for finding the answer to a question by supposition; and is founded upon the principle, that if the supposition be not true, the result obtained will differ from the result given in the question, in the same ratio in which the supposition itself differs from the truth.

2. It is divided into single and double position. Single position employs one supposition only, from which the true answer may be found by proportion according to the definition above given.

3. But the rule can only be applied when the quantities concerned in the question, bear some determinate ratio to the quantity sought; when this is not the case, the rule will not apply; and where it is the case, the answer may always be found by proportion alone, without any supposition at all.

Thus, to find a person's income, who, after spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of it, had £20 remaining; we suppose his income to have been £120; then, according to the conditions of the question, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of £120 $= 60 + 30 + 15 =$ £105, the part spent; and $120 - 105 =$ £15, the part remaining; but this remainder, according to the question, should be £20.

Hence, since the result thus obtained, differs from the result given in the question, in the same ratio in which the supposition differs from the truth, we find the true income by proportion in the ratio 15 : 20; thus 15 : 20 : : £120 : £160, the true income; for $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ of £160, $= 80 + 40 + 20 =$ £140; and £160 $- 140 =$ £20, the result given in the question.

But this result might have been obtained without any supposition at all; for $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4+2+1}{6} = \frac{7}{8}$; and $\frac{8}{8} - \frac{7}{8} = \frac{1}{8}$ the remainder, which by the question

is £20; consequently, as we know the value of $\frac{1}{8}$, we find the value of the whole in the ratio 1 : 8; thus 1 : 8 : : £20 : £160, the whole income, as before.

RULE.

Suppose any convenient number, and proceed with it according to the conditions of the question; if the result be not such as the question requires, say, as the result of the supposed number is to the result in the question, so is the supposed number to the true number.

EXAMPLES FOR PRACTICE.

1. A person planted a number of trees, of which $\frac{1}{3}$ were apple, $\frac{1}{4}$ pear, $\frac{1}{6}$ damascene, and the remainder, 48 in number, were cherry-trees; how many did he plant?

Ans. 192.

2. A person, after spending $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of his income, finds himself £60 in debt; what was his income?

Ans. £720.

3. A.'s age is $\frac{2}{3}$ of B.'s; and B.'s $\frac{3}{4}$ of C.'s, and their joint ages are 180 years; what is the true age of each?

Ans. A.'s 40, B.'s 60, and C.'s 80 years.

4. A person distributing a sum of money among several poor people, gave to the first $\frac{1}{2}$ of what he had, to the second $\frac{2}{3}$ of the remainder, to the third $\frac{3}{4}$ of what he had left, and to the fourth, the whole of the residue, which was 2*s*. 6*d*., what sum did he distribute?

Ans. He distributed 60*s*.

5. A gentleman bought a chaise, horse, and harness for £75; the chaise cost 5 times as much as the harness, and the horse 3 times as much as the chaise and harness; what was the price of each?

Ans. Harness, £3 „ 2*s*. 6*d*.; Chaise, £15 „ 12*s*. 6*d*.; Horse, £56 „ 5*s*.

6. A general, after an engagement, found that $\frac{1}{5}$ of his men were killed, $\frac{1}{10}$ wounded, $\frac{1}{15}$ taken prisoners, and that $\frac{1}{30}$ had deserted, so that he could muster only 1500 effective men; how many had he before the action?

Ans. 2500 men.

DOUBLE POSITION.

Double Position is a rule for finding, by two several suppositions, an answer to such questions as include quantities which bear no determinate ratio to the quantity sought; and is founded on the principle, that the errors resulting from the suppositions, bear to each other the same ratio as the differences between the several suppositions and the truth.

For instance, if it were proposed to find what income a person had who, after spending $\frac{3}{4}$ of it and £10 more, had £20 remaining; the answer could not be found by one single supposition, because, as it is uncertain what the income may be, it is impossible to determine the ratio which the quantity £10 bears to the quantity sought.

The answer, however, may be easily found by Proportion, without any supposition at all; for whatever the income may be, the quantity $\frac{3}{4}$ bears to it, the determinate ratio 3 : 4, and the £10 afterwards spent, and the £20 still remaining, together = £30, are evidently = the whole income $-\frac{3}{4}$, or $\frac{4-3}{4}=\frac{1}{4}$, consequently $1:4::30:120$, the income sought; for $\frac{3}{4}$ of £120 = £90, and £90 + 30 = £120.

To obtain the answer to this question by Double Position, we first suppose the income to be £160, then $\frac{3}{4}$ of

£160 = £120, and £120 + £10 = £130, the part spent, and £160 − £130 = £30, the remainder; but this remainder, according to the question, should be £20; consequently £30 − £20 = £10, the error resulting from this supposition.

Secondly, we suppose the income to be £140; then $\frac{3}{4}$ of £140 = 105, and £105 + £10, = £115, the part spent, and £140 − £115 = £25, the remainder; but this remainder, according to the question, should be £20; consequently £25 − £20 = £5, the error resulting from this second supposition.

Now each supposition obviously contains the truth, either increased or diminished by the quantity of error combined with it; and as we know from the previous solution, that the true income is £120, it is evident that the first supposition, £160, contains the truth, £120 + the difference £40, and the second supposition, £140, contains the truth, £120 + the difference £20.

And as from the principle on which the Rule of Double Position is founded, the errors resulting from the supposition will bear to each other the same ratio as the differences between the several suppositions and the truth, we have in this example,—the diff. 40 : the diff. 20 : : the error 10 : the error 5.

Here it is manifest that 40, the difference between the first supposition and the truth, and 5, the error resulting from the second supposition, are the extremes; and 20, the difference between the second supposition and the truth, and 10, the error resulting from the first supposition, are the means of proportional quantities, and consequently their products are equal.

Hence, if we multiply the first supposition by the second error, and the second supposition by the first error, those parts of the products which consist of the differences multiplied by the errors, will be equal; and if the errors are similar, or both too great or both too small, will vanish by subtraction; or if the errors be dissimilar, as when one is too great and the other too small, they will both vanish by addition.

Consequently, in either case, the difference or the sum of the products will consist only of the product of the

truth contained in each of the suppositions, multiplied, either by the difference or by the sum of the errors; whence it is evident that in the former case, if this product be divided by the difference of the errors, or in the latter by the sum of errors, the quotient must be the truth.

Thus, in reference to the preceding example:

$$\begin{array}{lcr}
\text{the 1st sup. } 160 = \text{truth } \overline{120 + \text{diff. } 40} \times 5 & = & \overline{600 + 200} \\
\text{the 2nd sup. } 140 = \text{truth } \overline{120 + \text{diff. } 20} \times 10 & = & \overline{1200 + 200} \\
\overline{5)} & & \overline{600 \quad\; *} \\
 & & 120 \text{ Truth.}
\end{array}$$

Here it is evident that the products of the differences 40 and 20, multiplied respectively by the errors 5 and 10, are both equal to 200, and consequently vanish by subtraction, leaving in the remainder, 600 only, the difference between 600 and 1200, the products of the truth in each supposition multiplied by the difference of the errors; and this difference, 600, divided by 5, the difference of the errors, gives the quotient, 120, which is the truth; or the sum 600 + 1200 = 1800, divided by 15, the sum of the errors, gives the quotient, 120, which is the truth, as before. Hence the

RULE.

1. Suppose any two convenient numbers, and proceed with them according to the conditions of the question; compare the results thus obtained with the result given in the question, and if greater or less the difference will be the error.

2. Multiply the first supposition by the second error, and the second supposition by the first error; and if the errors are similar, divide the difference of the products by the difference of the errors; but if the errors are dissimilar, divide the sum of the products by the sum of the errors, and the quotient will be the true number sought.

Ex.—A person, after spending $\frac{3}{4}$ of his income and £10 more, has £20 remaining; what was his income?

1.

1st Sup. income to be		£160
then $\frac{3}{4}$ of £160	= 120	
and £10 more	= 10	
part spent	=	130
remainder	=	30
which should be		20
the 1st error		£10

2.

2nd Sup. income to be		£140
then $\frac{3}{4}$ of £140	= 105	
and £10 more	= 10	
part spent	=	115
remainder	=	25
which should be		20
the 2nd error		£5

3.

1st Sup. 160×5 2nd er. =	800
2nd Sup. 140×10 1st er. =	1400
5)	600
the true income	£120

4.

Proof, inc.		£120
$\frac{3}{4}$ of 120	= 90	
and £10 more	= 10	
part spent	=	100
remainder		£20

This example is sufficiently explicit without any further illustration, than to remark, that the 1st sup., £160, multiplied by the 2nd error, £5, gives the product, £800; and the 2nd sup. £140, multiplied by the 1st error, £10, gives the product, £1400; and the difference of these products, £600, divided by the difference of the errors, £5, gives the quotient, £120, which is the true income.

EXAMPLES FOR PRACTICE.

1. A. is three times as old as B., but if 40 years be added to B.'s age he would be twice as old as A.; what is the age of each? *Ans.* A.'s age is 24, and B.'s age 8 years.

2. A labourer was hired on condition, that for every day he worked he should receive 20*d.*, and for every day he was idle he should forfeit 10*d.*; at the end of 300 days he received 10 guineas; how many days did he work, and how many days was he idle?
Ans. He worked 184 days, and was idle 116 days.

3. A person has two silver cups of unequal weight, and a cover weighing 10 oz.; if the cover be put on the 1st cup it will make its weight double that of the 2nd cup, and if put on the 2nd cup, will make its weight three times that of the 1st cup; what is the weight of each cup?
An. The 1st cup 6 oz., the 2nd cup 8 oz.

4. The head of a fish is 9 inches long, the tail is as long as the head and half the body, and the body is as long as the head and tail together; what is the length of the whole fish? *Ans.* 72 inches.

5. A person wishing to relieve a certain number of poor families, finds that if he gives each family 25*s.* he shall want £2,, 10*s.* more than he has, and if give each only 20*s.* he shall have £5 remaining; how many families had he to relieve? *Ans.* 30 families.

6. A gentleman has two horses, and a saddle worth £15; if the saddle be put on the first horse it will make his value 3 times that of the second, and if put on the second horse, will make his value twice that of the first; what was the value of each horse?

Ans. 1st horse £12, 2nd £9.

INVOLUTION.

1. A QUANTITY is said to be involved, when it is multiplied by itself any number of times; its several products are called powers, and these powers are denoted by indices written above them, which show the number of times the quantity occurs in the involution.

2. The quantity itself, which is the root, is called the first power; the quantity multiplied by itself once, is called the second power, or square; when multiplied by itself twice, the third power, or cube, and when multiplied by itself three times, the fourth power, or biquadrate, and so on.

Thus two is the first power, or root; $2 \times 2 = 4$, the second power, or 2^2; $2 \times 2 \times 2 = 8$, the third power, or 2^3; $2 \times 2 \times 2 \times 2 = 16$, the fourth power, or 2^4; &c. &c. Hence to involve a quantity to any power, we have only to multiply it by itself as many times, less 1, as is denoted by the index of the power required.

3. In the involution of a quantity to any high power, after having obtained a few of the powers, if we add together the indices of any of those powers, and multiply together the powers denoted by those several indices, the product will be the power denoted by the sum of those indices.

Hence to find the ninth power of 2, after having found the 2nd, 3rd, and 4th powers in the usual way, if we add together the indices, $2 + 3 + 4 = 9$, and multiply together the 2nd, 3rd, and 4th powers of 2, the product will be the 9th power of 2, or the power denoted by the sum of these indices, 9.

Thus $2^2 = 4, 2^3 = 8$, and $2^4 = 16$; consequently $4 \times 8 \times 16 = 512 = 2^9$; $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$; or $2^4 \times 2^4 \times 2^1 = 16 \times 16 \times 2 = 512 = 2^9$, the same result as before; for the sum of the indices of the powers multiplied, $4 + 4 + 1 = 9$.

4. If the quantity to be involved be a vulgar fraction, the power of the numerator written over the power of the denominator will be the power of the fraction; thus $(\frac{3}{4})^3 = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64}$.

5. If the quantity to be involved be a mixed number, the fractional part must be reduced to a decimal and annexed to the whole number, before it is involved; thus, $(2\frac{3}{4})^3 = 2{\cdot}75 \times 2{\cdot}75 \times 2{\cdot}75 = 20{\cdot}796875$.

RULE.

Multiply the given quantity by itself, as many times, less 1, as is denoted by the index of the power; or, having found some of the powers, add together as many of their indices as in their sum will be equal to the index of the power required, and the product of the powers corresponding to those indices will be the power required.

EXAMPLES FOR PRACTICE.

1. What is the 2nd power or square of 15? *Ans.* 225.

2. What is the 3rd power or cube of 25? *Ans.* 15625.

3. What is the biquadrate or 4th power of 127?
Ans. 260144641.

4. What is the 4th power of ·000125?
Ans. ·000000000000000244140625.

5. What is the 5th power of $\frac{3}{5}$? *Ans.* $\frac{243}{3125}$.

6. What is the 7th power of $1\frac{1}{2}$? *Ans.* 17·0859375.

7. What is the 12th power of 9? *Ans.* 282429536481.

8. What is the 15th power of $\frac{1}{2}$? *Ans.* $\frac{1}{32768}$.

The extreme labour of involving large numbers to a high power has led to the construction of artificial numbers called Logarithms, of which the addition, subtraction, multiplication, and division are respectively equivalent to the multiplication, division, involution and evolution of the natural numbers.

Thus, to obtain the 25th power of any natural number, we have only to multiply its corresponding logarithm by 25, and the product will be the logarithm of that power, and the natural number corresponding to this logarithm will be the 25th power of the given number.

Also, to extract the 25th root of a given number, we have only to divide its corresponding logarithm by 25, and the quotient will be the logarithm of that root, and the natural number corresponding to this logarithm will be the 25th root of the given number.

Tables of Logarithms will therefore be found of important use in abridging the labour of all calculations in which involution of numbers to a high power, or the evolution or extraction of their roots are concerned, and are therefore recommended to the notice of the student.

EVOLUTION.

1. Evolution, or the extraction of roots, is a method of finding the original number or root from which any power has arisen by involution; and which must be such a number as, multiplied by itself a certain number of times, will give that power as its product.

Thus 2 is the square root of 4; for $2 \times 2 = 4$; 3 is the cube root of 27, for $3 \times 3 \times 3 = 27$; 4, the biquadrate root of 256, for $4 \times 4 \times 4 \times 4 = 256$.

2. The roots of numbers are denoted by prefixing to the number the sign $\sqrt{}$ for the square root, and for any other root, the same sign with the index of the root written within or above it.

Thus $\sqrt{4}$ denotes the square root of 4; $\sqrt[3]{27}$, the cube root of 27; $\sqrt[4]{256}$, the biquadrate root of 256, and so on.

The roots are also denoted by fractional exponents, as $4^{\frac{1}{2}}$ denotes the square root of 4; $27^{\frac{1}{3}}$, the cube root of 27; $256^{\frac{1}{4}}$, the biquadrate root of 256, &c. &c.

3. The power of any number may be always accurately found by involution; but the root of any power cannot always be accurately found, as the square root of 8, or the cube root of 9; for no number multiplied by itself will be equal to 8, or multiplied by itself twice be equal to 9.

4. Numbers of which the roots cannot be accurately found are called Surds, and numbers of which the roots can be accurately determined are called Rationals: thus, $\sqrt{8}$ and $\sqrt[3]{9}$ are surds, and $\sqrt{9}$ and $\sqrt[3]{27}$ are rationals.

5. The method of extracting the root of any power is derived from an attentive consideration of the manner in which that power has arisen from its root by involution, and of the several parts of which it will be found to consist.

6. In general the power of any number of one single figure will consist of just as many figures as are denoted by its index; thus, the square of 9 or $9^2 = 81$ consists of

2 figures, the cube of 9 or $9^3 = 729$, consists of three figures, the biquadrate of 9, or $9^4 = 6561$, consists of 4 figures, and so on.

7. Hence is obtained a rule for separating any power of which the root is to be extracted, into periods each containing one figure of the root, of which the first figure is easily found by trial, and the others successively by a process peculiar to the power of which the root is to be extracted.

It will be sufficient for the purposes of arithmetic to explain this process as applicable to the square and cube roots only, the analysis of the higher powers being too complicated to be well exhibited in numbers.

EXTRACTION OF THE SQUARE ROOT.

If we involve the number 25 to the square, or second power, we shall have $25 \times 25 = 625$, the square or second power of 25; and if we examine of what this power consists, we shall find, as in the annexed scheme, that it contains $20 \times 20 = 400$, which is the square of 20; $20 \times 5 = 100$; $5 \times 20 = 100$; and $5 \times 5 = 25$, which is the square of 5.

$$
\begin{array}{rcrcl}
 & 20 & + & 5 & = 25 \\
 & 20 & + & 5 & = 25 \\
\hline
 & 100 & + & 25 & = 125 \\
400 + 100 & & & & = 50 \\
\hline
400 + 200 & & + & 25 & = 625 \\
\end{array}
$$

Here it is evident that the power 625 contains not only 400, which is the square of 20, and 25, which is the square of 5, but also 200, or twice the product of 20×5; consequently, after deducting from it the square of the root 20, which we find by trial, we shall have the remainder, 225, which contains not only the square of 5, the other figure of the root, but also 200, or twice the product of 20×5.

Consequently, if we divide this remainder by 20×2, the quotient will be 5, which is the second figure of the root; and, as the remainder also contains 25, which is the square

of 5, if we add to the divisor the figure 5, we shall have the whole divisor, $\overline{20 \times 2} + 5 = 40 + 5$, contained 5 times in the whole remainder, 225.

And now multiplying the divisor, $40 + 5$ by 5, we shall have the product, 225, which contains twice the product of $20 \times 5 = 200$, and also the square of $5 = 25$, by subtracting which from 225, we completely exhaust the power, and evolve the root 25. Hence the

RULE.

1. Separate the given number of which the root is to be extracted, into periods of two figures each, beginning at the right hand in whole numbers, and at the left hand in decimals.
2. Find by trial the nearest square root of the highest period, and place it in the quotient, as in Division; subtract the square of the figure thus found from the period whose root it expresses, and to the remainder, bring down the next lower period for a new dividend.
3. Double the figure of the root last found for a divisor, and find how often it is contained in the dividend exclusive of the place of units, and write the result in the next lower place of the quotient, and also in the next lower place of the divisor.
4. Multiply the whole of the divisor by the figure last placed in the quotient, and subtract the product from the dividend, and to the remainder bring down the next lower period for a new dividend.
5. Double the last figure of the preceding divisor, or, what is the same thing, double all the figures of the root already in the quotient, for a new divisor; find how often this divisor is contained in the dividend exclusive of the place of units, and write the result in the next lower place of the quotient and also of the divisor as before.
6. Continue this process till the roots of all the periods into which the given number was separated

have been found; and the several figures in the quotient, taken together in the order in which they arise, will be the square root of the given number, which will always consist of as many figures as there are periods, both integral and fractional.

Ex.—What is the square root of 549·9025?

	5, 49·90, 25 (23·45
	4
43)	149
3	129
464)	2090
4	1856
4685)	23425
	23425

Here, beginning at the right hand of the whole numbers, we mark off the periods 49 and 5; and at the left hand of the decimals, the periods 90 and 25. 2nd. We find by trial the nearest square root of the period 5 to be 2, which we write in the quotient, and subtracting its square from the period 5, we get the remainder, 1, to which we annex the next lower period, 49, and obtain the dividend, 149. 3rd. Doubling the root, 2, we get the divisor, 4, which is contained three times in the dividend, 149; we therefore write 3 in the next lower place of the quotient, and also of the divisor, making the whole divisor 43, and multiplying 43 by 3 we subtract the product, 129, from the dividend, 149, and get the remainder, 20, to which we bring down the next lower period, 90, for a new dividend. 4th. Doubling the last figure of the preceding divisor, 43, by adding 3 to it, we get the new divisor, 46, which, being contained 4 times in the dividend, 2090, we write 4 in the next lower place of the quotient, and also in the next lower place of the divisor, making the whole divisor 464; and proceeding with this as before, we get the next figure of the root, making the whole root 2345, which consists of just as many figures as there are periods; but as two of the periods are decimals, we mark off two decimal places in the quotient, and obtain the root 23·45.

EXAMPLES FOR PRACTICE.

1. Required the square root of 5184. *Ans.* 72.
2. What is the square root of 70225? *Ans.* 265.
3. Find the square root of 1522756. *Ans.* 1234.

4. What is the square root of 30712·5625?

Ans. 175·25.

5. Required the square root of 1524155677489.

Ans. 1234567.

6. Find the square root of ·000000015625.

Ans. ·000125.

7. What is the square root of $27\frac{3}{27}$?

Ans. 5·2068331, &c.

8. Required the square root of $\frac{625}{2401}$. *Ans.* $\frac{25}{49}$.

9. What is the square root of ·000000133225?

Ans. ·000365.

By the 47th Proposition of the First Book of Euclid, it is demonstrated, that in any right angled triangle the square of the side subtending the right angle, is equal to the sum of the squares of the sides containing the right angle.

In a right angled triangle, the side subtending the right angle is called the Hypothenuse; and the sides containing the right angle are called respectively the Base, and the Perpendicular.

From the Proposition referred to, it will be evident, that if, from the square of the hypothenuse, the square of the base be subtracted, the remainder will be the square of the perpendicular; or if the square of the perpendicular be subtracted, the remainder will be the square of the base.

Thus, if the hypothenuse of a right angled triangle be 55 feet, and the base 33 feet, then $55^2 - 33^2$, or $3025 - 1089 = 1936$ feet, the square of the perpendicular; and $\sqrt{1936} = 44$ feet, the length of the perpendicular.

Or, $3025 - 1936 = 1089$, the square of the base; and $\sqrt{1089} = 33$, the length of the base; also $1089 + 1936 = 3025$, the square of the hypothenuse; and $\sqrt{3025} = 55$, the length of the hypothenuse.

Hence it is evident that, from the principle above referred to, the following questions may be easily solved:—

10. What must be the length of a scaling-ladder to reach the battlement of a fort 33 feet high from the ground, the moat flowing at its base being 44 feet broad?

Ans. 55 feet.

11. What must be the length of a shore fixed at the distance of 18 feet from the base of a building to support a jamb 24 feet from the ground? *Ans.* 30 feet.

12. How long must a ladder be to reach the top of a wall, 41 feet 3 inches in height from the surface of a river 23 yds. 1 foot in breadth, which flows at its base?

Ans. 81 ft. 3 inches.

13. A shore, 25 feet in length, supported a jamb 20 feet in height from the ground; at what distance from the base of the building was it fixed? *Ans.* 15 feet.

14. A ladder, 100 feet long, is so placed against a building, which is 100 feet in height, as to reach within 20 feet from its summit; how far from the base of the building was the foot of the ladder placed? *Ans.* 60 feet.

15. The slant of a roof is 15 feet on each side, and the perpendicular height of the gable 9 feet; what is the breadth of the building? *Ans.* 24 feet.

16. A ladder, 50 feet long, is so placed as to reach a window on one side of the street 30 feet high, and by turning it over, without removing the foot, to reach a window 40 feet high on the opposite side of the street; what is the breadth of the street? *Ans.* 70 feet.

17. Two towers of a fort are 125 feet distant from each other, the one is 100 feet high, and the other 75 feet; at what distance from the base of each must a ladder 125 feet long be placed that, without removing the foot, it may reach the top of each tower?

Ans. 75 feet from the higher, and 100 feet from the lower tower.

18. Two trees, on a horizontal plane, are 120 feet distant from each other; the one tree is 100 feet, and the other 80 feet high; on what part of the plane must a person stand that his distance from the top of each tree may be the same as the distance of the tops of the trees from each other?

Ans. 69·2840824 feet from the higher, and 91·6515138 feet from the lower tree.

EXTRACTION OF THE CUBE ROOT.

$$
\begin{array}{rcl}
20+\ 5 & = & 25 \\
20+\ 5 & = & 25 \\
\hline
100+\ 25 & = & 125 \\
400+\ 100\qquad & = & 50 \\
\hline
400+\ 200+\ 25 & = & 625 \\
20+\ 25 & = & 25 \\
\hline
2000+1000+125 & = & 3125 \\
8000+4000+\ 500\qquad & = & 1250 \\
\hline
8000+6000+1500+125 & = & 15625
\end{array}
$$

If we involve 25 to the cube or 3rd power, by multiplying it twice by itself, we shall find, as in the annexed scheme, that the product contains, not only 8000, which is the cube of 20, and 125, which is the cube of 5, but also 6000, which is the product of 3 times the square of 20 multiplied by 5; and also 1500, which is the product of 3 times 20 multiplied by the square of 5; and from this analysis of the power we shall find a Rule for the extraction of its root.

Thus, to extract the cube root of 15625:—

$$
\begin{array}{lcll}
 & & 15{,}625\ (\ 25 & \\
 & & 8 & \\
2^2\times 3 = 12 & \text{Divisor)} & 7625 & \text{Resolvend.} \\
5^3 & = & 125 & \\
5^2\times 3\times 7 & = & 150 & \\
5\times 3\times 2^2 & = & 60 & \\
\hline
 & & 7625 & \text{Subtrahend.}
\end{array}
$$

Here, separating the given number 15625 into periods of 3 figures each, we find by trial the cube root of the first period 15 to be 2, which we write in the quotient for the first figure of the root. 2nd. Subtracting the cube of 2 = 8 from the first period 15, we get the remainder 7, to which we bring down the next lower period 625, making 7625, from which the remainder of the root is to be extracted, and which is therefore called the Resolvend. 3rd. As this resolvend contains 6000, which is the product of 3 times the square of 20 multiplied by 5, it is evident that if we divide the resolvend by 3 times the square of 20 the quotient will be 5, which we therefore write in the next lower place in the quotient for the second figure of the root. 4th. We now subtract from the resolvend, the cube of 5 = 125 which is contained in it, and as it contains also the square of 5 multiplied by 3 times 20 = 1500, we place under 125, the product of $5^2 \times$ by 3 times

$2 = 150$, which, written one place to the left, is equivalent to 1500, or $5^2 \times 20$; and as the resolvend contains also 3 times the square of 20 multiplied by $5 = 6000$, we place under the 150, the product of $5 \times 3 \times 2^2 = 60$, which, written two places to the left, is equivalent to 6000. And lastly, adding together these several quantities, we get their sum, 7625, which is called the Subtrahend; and subtracting this from the resolvend, completely exhaust the power 15625, and evolve its root 25.

It may be observed that the last number of the subtrahend may be always found by simply multiplying the preceding divisor by the last figure of the root, for the divisor always contains 3 times the square of all the preceding figures of the root. Hence the

RULE.

1. Separate the given number into periods of 3 figures each, beginning at the right hand in whole numbers, and at the left hand in decimals; find by trial the nearest cube root of the first period, and place it in the quotient for the first figure of the root.

2. Subtract the cube of the figure thus found from the period whose root it expresses; and to the remainder, bring down the next lower period for a resolvend.

3. Take 3 times the square of the figure, or if there be several, of all the figures in the quotient for a divisor, and find how often it is contained in a sufficient number of the first figures of the resolvend, and write the result in the quotient for the next figure of the root.

4. Under the resolvend write the cube of the figure last found; under this, the square of the last figure multiplied by 3 times all the preceding figures of the root; and under this quantity write the product of the last figure multiplied by 3 times the square of the preceding figures of the root, or, what is the same thing, the product of the last divisor multiplied by the last figure of the root, placing

each of the products one place to the left of the preceding, and their sum will be the subtrahend.

5. Subtract the subtrahend from the resolvend, and to the remainder bring down the next lower period of the given number for a new resolvend, with which proceed as before, till all the periods into which the given number was separated have been brought down; and the several figures in the quotient taken together, in the order in which they arise, will be the root of the given number.

Ex. 2.—To find the cube root of 34965783:—

	34,965,783 (327 Root.
	27
$3^2 \times 3 = 27$ Divisor)	7965 Resolvend.
$2^3 =$	8
$2^2 \times 3 \times 3 =$	36
$2 \times 3 \times 3^2$ or Divisor $27 \times 2 =$	54
	5768 Subtrahend.
$32^2 \times 3 = 3072$ Divisor)	2197783 Resolvend.
$7^3 =$	343
$7^2 \times 32 \times 3 =$	4704
$7 \times 3 \times 32^2$; or Div. $3072 \times 7 =$	21504
	2197783 Subtrahend.

EXAMPLES FOR PRACTICE.

1. What is the cube root of 421875? *Ans.* 75.
2. Required the cube root of 19683. *Ans.* 27.
3. Find the cube root of 1092727. *Ans.* 103.
4. Required the cube root of 28372625000. *Ans.* 3050.
5. What is the cube root of ·000001728? *Ans.* ·012.

6. Find the cube root of 43132·764843. *Ans.* 35·07.

7. Required the cube root of 27189441343.
Ans. 3007.

8. What is the cube root of $17\frac{2}{7}$? *Ans.* 2·5856, &c.

9. What is the cube root of $\frac{729}{4096}$? *Ans.* $\frac{9}{16}$.

10. Required the cube root of 12345·6789.
Ans. 23·112, &c.

The following is part of Horner's Rule for the extraction of the cube root, as given by Professor Young in his valuable treatise on the Theory of Equations, in which a full demonstration of the principle, and also additional abbreviations, may be found.

RULE II.

After separating the given number into periods and finding the root of the first period by trial, as in the common method which has been previously given,

1. Take 3 times the square of the first figure of the root for a trial divisor, and finding how often it is contained in the resolvend, write the result in the quotient for the next figure of the root.

2. Multiply the figure of the root last found, together with 3 times the preceding figure prefixed to it, by itself; and to the trial divisor add the product written two places to the right hand of it, and their sum will be the true divisor.

3. Multiply the true divisor by the last figure of the root, and, subtracting the product from the resolvend, bring down to the remainder the next lower period of the given number for a new resolvend.

4. To the preceding true divisor, and also to the quantity added to the trial divisor in producing it, add the square of the last figure of the root, and their sum will be the next trial divisor, from which the next figure of the root will be found as before.

5. From this last trial divisor find the next true divisor, by adding to it the product of the last

figure of the root, together with 3 times all the preceding figures prefixed to it, multiplied by itself, and written two places to the right hand of the trial divisor, as before.

6. Multiply this true divisor by the last figure of the root, and subtract the product from the resolvend, bringing down to the remainder the next lower period for a new resolvend, as before; and continue this process till all the periods have been brought down, and the several figures in the quotient will be the root.

Ex. 1.—To extract the cube root of 41063625.

	41,063,625 (345 Resolvend.
	27
$3^2 \times 3 =$ Trial divisor $= 27$)	14063 Resolvend.
$3 \times 3 = 9$; and $94 \times 4 = 376$	
True divisor 3076	12304 Subtrahend.
$4^2 =$ 16	
Trial divisor 3468)	1759625 Resolvend.
$34 \times 3 = 102$; $1025 \times 5 = 5125$	
True divisor 351925	1759625 Subtrahend.

Ex. 2. Required the cube root 77145562593.

	77, 145, 562, 593 (4257
	64
$4^2 \times 3 =$ Trial divisor 48)	13145 Resolvend.
$4 \times 3 = 12$; $122 \times 2 = 244$	
True divisor 5044	10088
$2^2 =$ 4	
Trial divisor 5292)	3057562
$42 \times 3 = 126$; $1265 \times 5 = 6325$	
True divisor 535525	2677625
$5^2 =$ 25	
Trial divisor 541875)	379937593 Resolvend
$425 \times 3 = 1275$; $12757 \times 7 = 89299$	
True divisor 54276799	379937593

EXAMPLES FOR PRACTICE.

1. Find by this method the cube root of 12167? *Ans.* 23.

2. What is the cube root of 156464? *Ans.* 54.

3. Require the cube root of 12326391. *Ans.* 231.

4. What is the cube root of 1754049816? *Ans.* 1206.

5. Find the cube root of 673373097125. *Ans.* 8765.

6. Required the cube root of ·00098611128? *Ans.* ·0462.

For further practice, find by this method the answers to the questions given under Rule I.

In the 33rd proposition of the 11th book of Euclid, it is demonstrated that similar solid parallelopipeds are to one another in the triplicate ratio of their like sides; and in the 18th of the 12th book, that spheres have to one another the triplicate ratio of that of their diameters.

By these principles the following questions may be solved; for the ratio of the cubes of the like sides in the one, and of the cubes of the diameters in the other, will determine their relative magnitudes respectively.

1. A cellar is 11 feet 5 inches in length, the same in breadth, and the same in depth; how many cubic feet of earth were dug out in its formation? *Ans.* $1488\frac{89}{1728}$ feet.

2. A cubical vessel, of which the side is 12 inches, will contain a certain number of pints; what must be the side of another to contain 8 times the number? *Ans.* 24 in.

3. The solid content of a block of marble is 389017 inches; what will be the side of a cube of equal solidity? *Ans.* 73 inches.

4. A ball of 4 inches diameter weighs 18 pounds; what will be the diameter of another weighing 114 lb.? *Ans.* 7·4006 inches.

5. What is the difference in cubic inches, between the cube of half a foot, and the half a cubic foot? *Ans.* 648 inches.

6. A cubic inch of glass is blown into the form of a globe that will hold 1 pint of wine; what is the thickness of the glass? *Ans.* ·0192 inch.

7. What is the internal length of the side of a cubical bin that contains $22\frac{1}{2}$ quarters of corn, supposing an imperial gallon to contain 2218·192 cubic inches?

Ans. 73·6322 inches.

8. If a brass cannon $11\frac{1}{4}$ inches in diameter weigh 1000 lb., what will be the weight of a similar cannon of which the diameter is 20·83 inches?

Ans. 5942·5697 lb.

9. If a ship of 300 tons burden be 44 feet in the length of the keel, what will be the length of the keel of a ship of 560 tons burden? *Ans.* 54·1761 feet.

10. Suppose the length of a ship's keel to be $41\frac{3}{4}$ yards, the breadth of the mid-ship beam $8\frac{1}{2}$ yards, and the depth of the hold 5 yards; required the dimensions of another of similar construction, which will carry 3 times the burden?

Ans. The keel, 180·28125 feet; the mid-ship beam, 36·05625 feet; and the depth of the hold, 21·63375 feet.

PROGRESSION.

Progression is a series of proportional quantities, in which every preceding term bears to the next succeeding term the same ratio throughout the whole series. Progression is of two kinds, arithmetical and geometrical.

1. Arithmetical progression is a series of quantities in arithmetical proportion, of which, as we have not hitherto treated, it may be necessary to give some account.

2. Referring to the definition of geometrical ratio and proportion already given, if for multiplication and division we substitute addition and subtraction, everything that has been said of the one will apply to the other.

3. Hence four quantities are in arithmetical proportion, if the difference between the first and second is the same as the difference between the third and the fourth; thus $3 : 5 :: 7 : 9$ are quantities in arithmetical proportion, for $5-3=2$, and $9-7=2$.

4. Hence also, in arithmetical proportion, the sum of

the extremes is equal to the sum of the means; thus if $3:5::7:9$, $3+9=12$, and $5+7=12$; and if from the sum of the means, either of the extremes be subtracted, the difference will be the other extreme; thus $\overline{5+7}-3=9$, and $\overline{5+7}-9=3$.

5. In any series of quantities in arithmetical progression, the sum of every two terms at an equal distance from the centre will be equal; if the number of terms be even, the whole series will divide into pairs, and if uneven, the central term will stand alone.

Thus in the series $2:4:6:8:10:12$, we have $2+12$, $=14$, $4+10=14$, and $6+8=14$, and in the series $2:4:6:8:10:12:14$, we have $2+14=16$, $4+12=16$, $6+10=16$, each of which is equal to twice the central term 8, or $8+8=16$.

6. An arithmetical mean between any two quantities is equal to half the sum of those quantities; thus the arithmetical mean between 6 and 10 is $=\overline{6+10}\div 2=8$; and taking this mean into the series, we have the progression $6:8:10$.

The most useful problems in arithmetical progression are the following, for the solution of which the principles given above will be found sufficient:—

PROBLEM I.

The two extremes, and the number of terms of an arithmetical progression being given, to find the ratio of the series or common difference between the terms.

1. As the last term of an arithmetical progression evidently arises from the continual addition of the common difference to the first term, it must consequently be greater than the first term by all that has been added to it in the formation of all the succeeding terms of the series.

2. Hence if the first term be subtracted from the last, the remainder must be the amount of all the additions that have been made to the first term; and as these additions have been made in equal portions, if we divide this amount by the number of the additions, the quotient will

be the amount of each separate addition, or the common difference between the terms.

3. But as the first term, being the root of the progression, does not arise from the addition of the common difference, it is evident that the number of additions will always be less by 1 than the number of the terms in the series; hence if we divide the difference between the first and the last term by the number of terms less 1, the quotient will be the ratio of the series, or the common difference required.

Ex.—The first term of an arithmetical progression is 3, the last term 24, and the number of terms 8; required the common difference.

Here, subtracting the first term 3 from the last term 24, we have $24 - 3 = 21$; and dividing this remainder 21 by the number of terms less 1, we have $21 \div 8 - 1$, or $21 \div 7 = 3$, the common difference; and the whole series 3 : 6 : 9 : 12 : 15 : 18 : 21 : 24. Hence the

RULE.

> Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference, or ratio of the progression.

EXAMPLES FOR PRACTICE.

1. The first term of an arithmetical progression is 3, the last term 27, and the number of terms 13; what is the common difference? *Ans.* 2.

2. The extremes are 3 and 48, and the number of terms 10; required the common difference. *Ans.* 5.

3. The extremes of an arithmetical progression are $\frac{1}{4}$ and $2\frac{1}{2}$, and the number of terms 10; required the common difference. *Ans.* $\frac{1}{4}$.

4. The first term is $3\frac{1}{2}$, the last term $47\frac{1}{2}$, and the number of terms 9; what is the ratio of the progression, or common difference? *Ans.* $5\frac{1}{2}$.

5. A person paying a sum of money in 12 weekly payments, and increasing each payment by an equal excess, paid the first week 9 shillings and the last week £4 „ 17*s.*; by how much did he increase each payment? *Ans.* 8*s.*

6. A person who habitually drank 6 quarts of wine per week, wishing to abolish the practice altogether by a regular decrease, in the 25th week drank none; by what quantity did he diminish his weekly allowance?

Ans. ½ of a pint.

PROBLEM II.

The two extremes and the common difference being given, to find the number of terms in the series.

This is precisely the converse of the preceding problem, for as the difference of the extremes divided by the number of terms less 1 gives the common differences; so the difference of the extremes divided by the common difference must give the number of terms less 1, to which, if we add 1, the sum will be the number of terms required. Hence the

RULE.

Divide the difference between the extremes by the common difference, and the quotient increased by 1 will be the number of the terms.

EXAMPLES FOR PRACTICE.

1. The extremes of an arithmetical progression are 5, and 125, and the common difference 5; required the number of terms. *Ans.* 25.

2. The extremes of a decreasing series are 10 and 5, and the common difference ⅕; required the number of terms. *Ans.* 26.

3. The extremes are 3½ and 64, and the common difference 5½; required the number of terms. *Ans.* 12.

4. A person travelling 5 miles on the first day of his journey, and increasing the distance by 5 miles daily, travelled on the last day 75 miles; for how many days did he travel? *Ans.* 15 days.

5. A person distributing a sum of money among a certain number of poor persons, gave to the first 30*s.*, to the second 27*s.* 6*d.*, decreasing each succeeding donation by 2*s.* 6*d.*, which was the amount of his last donation; how many persons did he relieve?

Ans. 12 persons.

6. The first term of a decreasing series in arithmetical progression is 24, the last term 20, and the common difference $\frac{1}{20}$; required the number of terms. *Ans.* 81.

PROBLEM III.

The first term, and the common difference being given, to find the last or any assigned term of the series.

As every term of the series except the first arises from the addition of the common difference to the first term, it is evident that if the common difference be multiplied by the number of terms less 1, the product will be the difference between the first term and the last.

Consequently if this product be added to the first term in an increasing, or subtracted from it in a decreasing, series, the sum in the former case, or the remainder in the latter, will be the last term of the series. *Note.*—The particular term required may always be regarded as the last: hence the

RULE.

Multiply the common difference, by the number of terms less 1, and add the product to the first term in an increasing series, or subtract it from the first term in a decreasing series, and the sum or the remainder will be the term required.

EXAMPLES FOR PRACTICE.

1. The first term of an arithmetical progression is 5, and the common difference 3; required the 15th term of the series. *Ans.* 47.

2. The first term of a series in arithmetical progression is 1, and common difference 5; required the 21st term. *Ans.* 101.

3. The first term of a decreasing series is $12\frac{1}{2}$, and the common difference $\frac{1}{2}$; required the 25th term of the series. *Ans.* $\frac{1}{2}$.

4. A debt was discharged in 12 weeks, by paying 15*s.* the first week, and increasing each succeeding payment by 5*s.*; required the amount of the last payment. *Ans.* £3 „ 10*s.*

PROBLEM IV.

The first term, the last term, and the number of terms, or the first term, the common difference, and the number of terms being given; to find the amount of all the terms, or the sum of the series.

From either of the above data, we can complete the series by the preceding problems; and this problem is nothing more than adding together the several terms in order to find their sum.

But when there are many terms in the series, the process by actual addition would be tedious; if therefore we write under the given series, the same in an inverted order, and add together every two terms, we shall obtain a series in which all the terms will be equal, and consequently of which the sum may be found by multiplication.

Thus, if under the series we write the same in an inverted order, and add both together, as in the annexed scheme, we shall have the series 16, 16, 16, 16, 16, 16, in which all the terms are equal; consequently, we have only to multiply one of these terms by 6, the number of terms, and we shall have $16 \times 6 = 96$, the sum of this series.

3	5	7	9	11	13
13	11	9	7	5	3
16	16	16	16	16	16

But it is evident that this last series is twice as great as the given series, 3, 5, 7, 9, 11, 13; for it is the sum of two of them added together; consequently, if we divide the sum of this last series by 2, we shall have $96 \div 2 = 48$, the sum of the given series, 3, 5, 7, 9, 11, 13; hence the

RULE.

Add the first term to the last, and multiply their sum by the number of terms; and the product divided by 2 will be the sum of the series; or multiply the sum of the extremes by half the number of terms, and the product will be the sum of the series.

EXAMPLES FOR PRACTICE.

1. The first term of an arithmetical progression is 3,

the last term 29, and the number of terms 15; required the sum of the series. *Ans.* 240.

2. The first term is 5, the common difference 4, and the number of terms 13; what is the sum of the series?

Ans. 377.

3. The first term of a decreasing series is 10, the last term 1, and the number of terms 19; what is the sum of the series? *Ans.* $104\frac{1}{2}$.

4. A person distributing money among 25 persons, diminishing each donation by an equal sum, gave to the first 35*s.*, and to the last 5*s.*; what sum did he distribute?

Ans. £25.

5. How many miles did that person travel, who, performing 15 miles the first day, and gradually increasing each day's journey by an equal addition, travelled 75 miles on the 13th day? *Ans.* 585 miles.

6. If 100 stones be placed in a straight line of 1 yard from each other, and the first 1 yard from a basket; what distance will that person travel who gathers them one by one into the basket? *Ans.* 5 miles 1300 yds.

PROBLEM V.

To find any number of arithmetical means between two given numbers.

This problem is simply finding the intermediate terms of an arithmetical progression, of which the two given numbers are the extremes; and which, together with the number of means, form the whole of the series.

Now, as all the intermediate terms arise from the addition of the common difference to the first term, we have only by Problem 1 to find the common difference of the series and add it to the first, and every succeeding term, till we obtain the number of means required.

Thus, to find 3 arithmetical means between the numbers 10 and 22; the two extremes 10 and 2, and the three means required, give us 5 for the number of terms in the series.

Hence, by Problem 1, we have $\overline{22 - 10} \div \overline{5 - 1} = 12 \div 4 = 3$, the common difference; and $10 + 3 = 13$, the first mean; $13 + 3 = 16$, the second mean; and

$16 + 3 = 19$, the third mean; and taking these means into the progression, we have the series 10 : 13 : 16 : 19 : 22; hence the

RULE.

Find the common difference of an arithmetical progression of which the two given numbers are the extremes, and, together with the required number of means, will be the number of the terms.

Add the common difference to the first term and to every succeeding term; or, if the series be decreasing, subtract it till you have obtained the proper number of intermediate terms; and the several sums in the one case, and the several remainders in the other, will be the arithmetical means required.

If one mean only be required, it will always be equal to half the sum of the given numbers; thus, the arithmetical mean between 4 and $12 = \overline{4 + 12} \div 2 = 16 \div 2 = 8$; and the mean between 5 and $13 = \overline{5 + 13} \div 2 = 18 \div 2 = 9$; whence the series 4 : 8 : 12, and 5 : 9 : 13.

EXAMPLES FOR PRACTICE.

1. Find 2 arithmetical means between the numbers 5 and 17. *Ans.* 9 and 13.

2. Required 3 arithmetical means between the numbers 15 and 75. *Ans.* 30, 45, and 60.

3. Find 1 arithmetical mean between the numbers 24 and 100. *Ans.* 62.

4. Required 5 arithmetical means between the numbers 7 and 27. *Ans.* $10\frac{1}{3}$, $13\frac{2}{3}$, 17, $20\frac{1}{3}$, and $23\frac{2}{3}$.

5. A person distributing money among 7 poor persons, gave 4*s.* to the first, and, increasing every succeeding donation by an equal sum, gave 40*s.* to the last; how much did each of the others receive?

Ans. The 2nd, 10*s.*; 3rd, 16*s.*; 4th, 22*s.*; 5th, 28*s.*; and the 6th, 34*s.*

6. Find 7 arithmetical means between the numbers 1 and 2. *Ans.* $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{5}{8}$, $1\frac{3}{4}$, and $1\frac{7}{8}$.

GEOMETRICAL PROGRESSION.

1. Geometrical Progression is a continued series of quantities in Geometrical Proportion, in which every preceding term bears to the next succeeding term the same ratio throughout the whole series.

Thus, 1 : 2 : 4 : 8 : 16 : 32, is an increasing geometrical progression, of which every preceding term bears to the next succeeding term the ratio of $1 : 2 = \frac{1}{2}$; and 32 : 16 : 8 : 4 : 2 : 1, is a decreasing geometrical progression, in which every preceding term bears to the next succeeding the ratio $2 : 1 = 2$, throughout the whole series.

In these progressions, it is usual, and perhaps more convenient, to consider the constant multiplier in an increasing series, and the constant divisor in a decreasing series, as the ratio of the progression; and in this view, the ratio of both the progressions given above will be 2.

Hence, 2 is the ratio of the increasing series in which every succeeding term is the product of the preceding term multiplied by 2; and also the ratio of the decreasing series, in which every succeeding term is the quotient of the preceding term divided by 2.

2. In any series of numbers in geometrical progression consisting of an even number of terms, the product of the two extreme terms is equal to the product of the two mean terms, and also to the product of any two terms equally distant from the centre; and in a series consisting of an uneven number of terms, these products are all equal to the square of the mean or central term.

Thus, in the series, 1 : 2 : 4 : 8 : 16 : 32, $1 \times 32 = 2 \times 16 = 4 \times 8 = 32$; and in the series 1 : 2 : 4 : 8 : 16, $1 \times 6 = 2 \times 8 = 4 \times 4 = 16 = 4^2$; hence, a geometrical mean between any two numbers is equal to the square root of their product; thus, a geometrical mean between 2 and 8 $= \sqrt{2 \times 8} = \sqrt{16} = 4$.

The most useful problems in geometrical progression, as applying to arithmetical purposes, are the following:—

PROBLEM I.

The first term, the ratio, and the number of terms being given; to find the last, or any assigned term of the series.

As every succeeding term, in a geometrical progression, arises from the multiplication or division of the next preceding term by the ratio, it is evident that the second term arises from the multiplication or division of the first term by the ratio; the third term from the multiplication or division of the second term by the ratio, or, what is the same thing, from the multiplication or division of the first by the second power of the ratio, and so on throughout the series.

Consequently, to find the last, or any assigned term of the series, we have only to multiply or divide the first term by the ratio involved, to a power of which, the index is less by 1 than the index of the term required, and the product in an increasing series, or the quotient in a decreasing series, will be the term required; hence the

RULE.

Involve the ratio of the progression to a power of which the index is less by 1 than the index of the required term; and in an increasing series multiply, or in a decreasing series divide, the first term by this power of the ratio, and the product or quotient will be the term required.

EXAMPLES FOR PRACTICE.

1. Required the fourth term of a geometrical progression, of which the first term is 5, and the ratio 3.
Ans. 135.

2. Required the fifth term of a decreasing series in geometrical progression, of which the first term is 1280, and the ratio 4. *Ans.* 5.

3. What is the last term of a series of which the first term is 3, the ratio 2, and the number of terms 13?
Ans. 12288.

4. What is the last term of a decreasing series, of which the first term is 45927, the ratio 3, and the number of terms 9? *Ans.* 7.

5. The first term is 64, the ratio $\frac{1}{2}$, and the number of terms 15; required the last term of the series. *Ans.* $\frac{1}{256}$.

6. A labourer agrees to work for 21 days, commencing

at one farthing for the first day and doubling his wages daily, on condition of receiving only the last day's wages; what sum did he receive? *Ans.* £1042,, 5*s*. 4*d*.

PROBLEM II.

The first term, the last term, and the ratio; or the first term, the number of terms, and the ratio being given, to find the sum of the series.

This problem might be solved by simply adding together all the terms of the series; but when the number of terms is great, the finding of their sum by actual addition would be so tedious, that some shorter method becomes requisite.

1. For this purpose, if we multiply every term of the given series by the ratio, we shall have a new series, of which every term, and consequently the sum, will be just as many times greater than the sum of the given series, as the ratio or multiplier is greater than 1.

2. If from this new series, we now subtract the given series, it is evident that the remainder will be just as many times greater than the sum of the given series, as the ratio less 1 is greater than 1.

3. Consequently, if we divide this remainder by the ratio less 1, the quotient will be the sum of the given series; as will be manifest in the following example.

Ex.—To find the sum of the series 5 : 15 : 45 : 135 : 405.

1. Here, multiplying each term of the given series by the ratio 3, we get the new series 15 : 45 : 135 : 405 : 1215, which is evidently 3 times as great as the given series, or as many times greater, as the ratio 3 is greater than 1.

	5 :	15 :	45 :	135 :	405			
					×3			
15 :	45 :	135 :	405 :	1215				
15 :	45 :	135 :	405 :	5				
*	*	*	*	1210				

$$\frac{1210}{3-1=2}=605$$

2. Subtracting from the last series, the given series 5 : 15 : 45 : 135 : 405, of which, merely for convenience, we write the first term 5 under the last term of the new series, we find in both, the terms 15, 45, 135, and 405, which in the subtraction cancel each other, leaving only in

the remainder 1210, the difference between 5, the first term of the given series, and 1215 the last term of the new series, which is the product of the last term of the given series 405 multiplied by the ratio 3. 3rd. As from the new series of which the sum, as we have seen, was 3 times as great as that of the given series, we have subtracted the given series, it is evident that the remainder is $3-1$ times, or just twice as great as the sum of the given series: consequently if we divide this remainder 1210 by 2, or by the ratio $3-1=2$, the quotient 605 will be the sum of the gi en series; hence the

RULE.

Multiply the last term of the given series by the ratio: from the product subtract the first term, and the quotient of the remainder, divided by the ratio less 1, will be the sum of the series.

EXAMPLES FOR PRACTICE.

1. The first term of a geometrical progression is 3, the last term 49152, and the ratio 2; required the sum of the series. *Ans.* 98301.

2. The first term is 5, the number of terms 15, and the ratio 4; what is the sum of the series? *Ans.* 1789569705.

3. The first term of a geometrical progression is 1342177280, the last term 1280, and the ratio 4; what is the sum of the series? *Ans.* 1789569280.

4. The first term of a geometrical progression is 1024, the last term 59049, and the ratio $1\frac{1}{8}$; what is the sum of the series? *Ans.* 175099.

5. A person selling a horse in whose shoes were 32 nails, agreed to receive as the price, the amount of all the nails, valuing the first at one farthing, and doubling the value of each nail to the 32nd; what did he receive for the horse? *Ans.* £4473924 „ 5*s.* $3\frac{3}{4}$*d.*

5. How many ships of 1000 tons burden would be required to export the last year's produce of one grain of wheat, supposing each grain to have produced annually 20 grains for 21 years, that 8000 grains would be equal to 1 pint, and 40 bushels equal to 1 ton? *Ans.* 5120,000000,000000.

PROBLEM III.

To find one or more geometrical means between any two given numbers.

1. As every term of a geometrical progression, except the first, arises from the multiplication of the first term by the ratio; and as the required number of means, are only the intermediate terms of a series of which the two given numbers are the extremes, it is evident that if we find the ratio of the progression, and multiply the first term and every resulting product by it, we shall obtain the number of means required.

2. To find the ratio of the progression, we have only to consider that the last term of the series is the product of the first term multiplied by the ratio, involved to a power of which the index is less by 1 than the index of the last term.

3. Hence if 5 means are required, these 5 means, together with the two given numbers, which are the extremes, will give 7 for the number of terms in the whole series, or 7 as the index of the last term.

4. And as the last term is always the product of the first term multiplied by the ratio, involved to a power of which the index is less by 1 than the index of that term, it is evident that if we divide the last term by the first, the 6th root of the quotient will be the ratio of the series; hence the

RULE.

1. Divide the greater of the two given numbers by the less, and from the quotient extract a root of which the index will be less by 1 than the index of the last term, or, what is the same thing, greater by 1 than the required number of means, and it will be the ratio of the series.
2. Multiply the first term, and every succeeding product, if the series be increasing; or divide the first term, and every succeeding quotient, if the series be decreasing, by the ratio till you have obtained the number of means required.

Ex.—Find 2 geometrical means between the numbers 5 and 16875?

Here dividing the last term 16875 by the first term 5, we get the quotient 3375, and extracting the cube or 3rd root of this quotient, we get 15 for the ratio of the progression, by which, multiplying the 1st term, we get 75 for the 1st mean, and multiplying the 1st mean by the ratio 15, we get 1125, the 2nd mean; and thus complete the series of the progression, 5 : 75 : 1125 : 16875.

$$16875 \div 5 = 3375$$
$$\sqrt[3]{3375} = 15 \text{ ratio}$$
$$5 \times 15 = 75 \text{ 1st mean}$$
$$75 \times 15 = 1125 \text{ 2nd mean}$$

EXAMPLES FOR PRACTICE.

1. Find one geometrical mean between the numbers 6 and 54. *Ans.* 18.

2. Find two geometrical means between the extremes 9 and 243. *Ans.* 27 and 81.

3. Find three geometrical means between the numbers 6 and 1536. *Ans.* 24, 96, and 384.

4. Find four means between the extremes 9 and 2187. *Ans.* 27, 81, 243, and 729.

5. A person, increasing his daily journey in an equal ratio, travels the 1st day $10\frac{2}{3}$ miles, and on the 7th day $121\frac{1}{2}$ miles; how many miles did he perform on each of the other days?

Ans. 2nd day 16, 3rd day 24, 4th day 36, 5th day 54, and on the 6th day 81 miles.

6. Find seven geometrical means between the extremes 1 and 390625.

Ans. 5, 25, 125, 625, 3125, 15625, and 78125.

COMPOUND INTEREST.

1. When the interest of a sum of money, instead of being paid yearly, half-yearly, or quarterly, as it may become due, is added to the principal; the interest which arises from this amount is called Compound Interest.

2. It will be unnecessary to give any example for practice in Compound Interest by the usual method, the calculation being precisely the same as in Simple Interest.

3. It will be sufficient to observe, that in Compound Interest, the interest for the 1st year being added to the principal, the amount is the principal for the 2nd year; the interest for the 2nd year, in like manner, being added to the principal for that year, the amount will be the principal for the 3rd year, and so on.

4. But when the number of years is great, this mode of calculation, requiring as many distinct sums in Simple Interest, as there are years, becomes tedious; consequently, a more expeditious method is derived from the principle of geometrical progression, which will equally apply to the discounting of a sum of money for any number of years at compound interest.

5. The interest of any principal at 5 per cent. for 1 year is $\frac{5}{100} = \frac{1}{20}$ of itself, and the amount, consequently, $\frac{21}{20}$; the interest of this amount, at the end of the 2nd year, will, in like manner, be $\frac{1}{20}$ of itself, and the amount, $\frac{21}{20} + \frac{1}{20}$ of $\frac{21}{20} = \frac{21}{20} \times \frac{21}{20} = \left(\frac{21}{20}\right)^2$; hence, $\frac{21}{20}$ is the ratio of a geometrical progression, of which the number of terms will be equal to the number of years, and the last term will be the amount of the given principal, at 5 per cent. for the time.

6. In this series, the ratio, $\frac{21}{20}$, is calculated at 5 per cent.; but the ratio, at any other rate, may be easily found; thus, at $2\frac{1}{2}$ per cent., any principal will gain

$\frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$ of itself; at $7\frac{1}{2}$ per cent., $\frac{7\frac{1}{2}}{100} = \frac{15}{200}$ $= \frac{3}{40}$ of itself; consequently, $\frac{41}{40}$ = the amount at $2\frac{1}{2}$, and $\frac{43}{40}$ = the amount at $7\frac{1}{2}$ per cent., will be the ratio of the progressions respectively.

Hence, to find the amount of any principal for any number of years at compound interest; we have only to make its amount for 1 year at the given rate per cent. the ratio of a geometrical progression, of which the number of terms will be equal to the number of years; and the last term of the series will be the amount required.

In order to keep the terms of the series as small as possible, if we make the amount of £1 for 1 year at the given rate per cent., the ratio of the progression, and multiply the last term of the series, which is the amount of £1, by the given principal, we shall obtain the same result.

Ex.—To find the amount of £500 for 5 years at 5 per cent. compound interest.

Here, making $\frac{21}{20}$ the amount of £1 for 1 year at the given rate per cent., the ratio of the progression; we have the series $\frac{21}{20} : \left(\frac{21}{20}\right)^2 : \left(\frac{21}{20}\right)^3 : \left(\frac{21}{20}\right)^4 : \left(\frac{21}{20}\right)^5$, in which the number of terms is equal to the number of years; and of which the last term, $\left(\frac{21}{20}\right)^5$ is the amount of £1 for the given time.

2. As this last term is involved to the 5th power, its value $= \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = \frac{4084101}{3200000}$; and multiplying this last term, which is the amount of £1, by the given principal, £500, we have $\frac{4084101 \times 500}{3200000} = \frac{2042050500}{3200000}$ $= 638{\cdot}14078125$ = £638 „ 2*s.* $9\frac{3}{4}$*d.* and $\frac{15}{100}$ = the amount of £500 for 5 years at 5 per cent. compound interest.

The amount of £1 for 1 year expressed by a decimal fraction, $= 1{\cdot}05$, might have been taken as the ratio of

the progression, and we should have had the series, $1{\cdot}05 : 1{\cdot}05^2 : 1{\cdot}05^3 : 1{\cdot}05^4 : 1{\cdot}05^5$, of which the last term $= 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 = 1{\cdot}2762815625$ = the amount of £1 for the time; and multiplying this quantity by the given principal, £500, we have $1{\cdot}2762815625 \times 500 = £638{\cdot}14078125 = £638$ „ 2*s.* $9\frac{3}{4}d.\ \frac{15}{100}$ the amount of £500, as before.

When the last term of the series contains a high power of the ratio, as will always be the case when the number of years is great, its value may be easily found by the help of logarithms.

RULE.

Find the amount of £1 for 1 year at the given rate per cent., and involve it to a power of which the index shall be equal to the number of years; multiply this power of the ratio by the given principal, and the product will be the amount required.

If the interest be payable half-yearly or quarterly, find the amount of £1 for a half-year or for a quarter of a year, and involve it to a power of which the index shall be equal to the number of half-years or quarters in the given time, and proceed as before.

If the interest only be required, subtract the given principal from the amount, and the remainder will be the interest.

EXAMPLES FOR PRACTICE.

1. What will be the amount of £350 for 5 years at 5 per cent., compound interest? *Ans.* £446 „ 13*s.* $11\frac{1}{2}d.$

2. What is the amount of £750 for 7 years at 5 per cent., compound interest? *Ans.* £1055 „ 6*s.* 6*d.*

3. What is the compound interest of £125 for $4\frac{1}{2}$ years at 4 per cent., the interest being payable half-yearly? *Ans.* £24 „ 7*s.* $8\frac{1}{2}d.$

4. What is the amount of £875 for $3\frac{3}{4}$ years at 6 per cent. compound interest, the interest being payable quarterly? *Ans.* £1093 „ 19*s.* $0\frac{3}{4}$*d.*

5. What is the compound interest of £375 „ 10*s.*, for 25 years, at 4 per cent.? *Ans.* £625 „ 10*s.* $5\frac{1}{2}$*d.*

6. What is the amount of £361 „ 17*s.* 6*d.* for 37 years, at 5 per cent. compound interest? *Ans.* £2200 „ 14*s.* $1\frac{1}{2}$*d.*

DISCOUNT.

The present worth of any sum of money due any number of years hence at Compound Interest, may in the same manner be found by making the present worth of £1 for 1 year, at the given rate per cent., the ratio of a geometrical progression, of which the number of terms shall be equal to the number of years for which the given sum of money is to be discounted.

Now, as at 5 per cent. the amount of £100 for 1 year is £105, it is evident that the present worth of £105 for the same time, and at the same rate per cent., must be £100. Hence the amount of £1 for 1 year, at 5 per cent., will be equal to $\frac{105}{100} = \frac{21}{20}$ £., and the present worth of £1 for the same time $= \frac{100}{105} = \frac{20}{21}$.

Consequently, if we invert the ratio of the progression by which we find the amount, we shall have the ratio of a progression, of which the last term will be the present worth of any given sum of money due any number of years hence at compound interest.

Ex.—To find the present worth of £500, due 5 years hence, discounting at 5 per cent., compound interest.

Here makin g $\frac{20}{21}$, the present worth of £1 for 1 year, at 5 per cent., the ratio of a geometrical progression, con-

sisting of 5 terms equal to the number of years for which the sum of money is to be discounted; we have the series, $\frac{20}{21} : \left(\frac{20}{21}\right)^2 : \left(\frac{20}{21}\right)^3 : \left(\frac{20}{21}\right)^4 : \left(\frac{20}{21}\right)^5$, of which the last term, $\left(\frac{20}{21}\right)^5$, will be the present worth of £1 for the given time.

2. As this last term is involved to the fifth power, its value is $= \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} = \frac{3200000}{4084101}$; and multiplying this quantity, which is the present worth of £1, by the given sum of money, £500, we have $\frac{3200000 \times 500}{4084101} = \frac{1600000000}{4084101} =$ £391 „ 15*s.* 3*d.* $\frac{13}{100}$, the present worth of £500, due 5 years hence, at compound interest. Hence the

RULE.

1. Find the present worth of £1 for 1 year, at the given rate per cent., and involve it to a power, of which the index shall be equal to the number of years for which the sum is to be discounted; multiply this power by the given sum, and the product will be the present worth required.
2. If the discount only is required, find the present worth of the given sum, as before; and subtracting this present worth from the given sum, the remainder will be the discount.

EXAMPLES FOR PRACTICE.

1. What is the present worth of £350 for 5 years, discounting at 5 per cent., compound interest?
Ans. £274 „ 4*s.* 8*d.* $\frac{1}{5}$.

2. What is the present worth of £1500 for 5 years, discounting at 5 per cent., compound interest?
Ans. £1175 „ 5*s.* 9$\frac{1}{4}$*d.*

3. What is the discount of £173 „ 15*s.* 5*d.* for 9 years, at 4 per cent., compound interest? *Ans.* £48 „ 15*s.* 5*d.*

4. What is the present worth of £1055 ,, 6*s*. 6*d*. due 7 years hence, discounting at 5 per cent., compound interest? *Ans*. £750.

5. What is the present worth of £375 ,, 10*s*. for 15 years, discounting at $7\frac{1}{2}$ per cent., compound interest? *Ans*. £123 ,, 13*s*. 1*d*.

6. What is the present worth of £1001 for 25 years, discounting at 4 per cent., compound interest? *Ans*. £375 ,, 10*s*.

ANNUITIES.

1. An annuity is a sum of money payable yearly, half-yearly, or quarterly, either for a certain number of years, or for life, or for ever.

2. Annuities are either in possession or in reversion; the former are such as are in actual course of payment; the latter such as will not commence till after a certain period of time has elapsed.

3. When an annuity, instead of being paid as it becomes due, is withheld for a certain time, it is said to be in arrears; and the sum to which the claimant is entitled, both for the principal and the interest, for the time it has been forborne, is called the amount.

4. The sum paid for the purchase of an annuity, or given as an equivalent for the discontinuance of the annual payment, is called the present worth.

The most important problems are, 1st, to find the amount of an annuity which has been forborne; and, 2nd, to find the present worth of an annuity either in possession or in reversion; in both of which the calculation is made on the principle of compound interest.

PROBLEM I.

To find the amount of an annuity forborne for any number of years at compound interest.

In finding the amount of any principal at compound interest, we have anticipated the calculation of the amount of an annuity; the only difference is, that in the former, the last term of the progression is the amount of the principal; and in the latter, not the last term only but the sum of all the terms is the amount of the annuity.

As there is no interest due on the annuity for the first year, the first term of the progression will be the annuity itself, the second term its amount for 1 year, and so on throughout the series; or, to keep the terms of the series as small as possible, £1 will represent the annuity, and the amount of £1 for 1 year at the given rate per cent., the ratio of the progression.

Thus, to find the amount of an annuity of £350, forborne for 5 years, at 5 per cent., we shall have the series $1 : \left(\frac{21}{20}\right) : \left(\frac{21}{20}\right)^2 : \left(\frac{21}{20}\right)^3 : \left(\frac{21}{20}\right)^4$; and the sum of this series, multiplied by 350, will be the amount of the given annuity, £350.

To find the sum of this series, we multiply the last term $\left(\frac{21}{20}\right)^4$ by the ratio $\frac{21}{20}$, and get the product $\left(\frac{21}{20}\right)^5$; from which, subtracting the first term, 1, we have $\left(\frac{21}{21}\right)^5 - 1 = \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = \frac{4084101}{3200000} - \frac{3200000}{3200000} = \frac{884101}{3200000}$; and lastly,

Dividing this remainder by the ratio less 1, or $\frac{21}{20} - \frac{20}{20} = \frac{1}{20}$, we have $\frac{884101}{3200000} \div \frac{1}{20} = \frac{884101}{3200000} \times \frac{20}{1} = \frac{1768202}{3200000}$ the sum of the series; and multiplying this sum by 350, we have $\frac{1768202}{320000} \times \frac{350}{1} = \frac{6188707}{3200} =$

£1933 ,, 19*s*. 5*d*.$\frac{1}{40}$ the amount of the given annuity £350. Hence the

RULE.

1. Make the amount of £1 for 1 year, at the given rate per cent., the ratio of a geometrical progression of which the first term is £1, and the number of terms equal to the number of years for which the annuity has been forborne.
2. Find the sum of the series, and it will be the amount of an annuity of £1 for the time; and multiply this sum by the given annuity, and the product will be the amount required.

EXAMPLES FOR PRACTICE.

1. What is the amount of an annuity forborne for 4 years, at 5 per cent., compound interest? *Ans.* £323 ,, 5*s*. 2½*d*.

2. What is the amount of an annuity of £150, forborne 7 years, at 4 per cent.? *Ans.* £1184 ,, 14*s*. 10¼*d*.

3. What is the amount of an annuity of £500 ,, 10*s*., forborne 10 years, at 3 per cent., compound interest? *Ans.* £5731 ,, 18*s*. 9½*d*.

4. What is the amount of an annuity of £250 ,, 10*s*., forborne 15 years, at 3½ per cent.? *Ans.* £4833 ,, 11*s*. 4¼*d*.

5. What is the amount of an annuity of £125 ,, 15*s*. for 35 years, at 5 per cent.? *Ans.* £11357 ,, 15*s*. 6¾*d*.

6. What is the amount of an annuity of £375 ,, 17*s*. 6*d*., forborne for 50 years, at 6 per cent.? *Ans.* £22029 ,, 4*s*. 8½*d*.

PROBLEM II.

To find the present worth of an annuity for any number of years at compound interest.

As in finding the amount of an annuity in the preceding problem, we made the amount of £1 for 1 year at the

given rate per cent. the ratio of a geometrical progression; so, to find the present worth of an annuity, we have only to make the present worth of £1 for 1 year at the given rate per cent. the ratio of the progression, and in every other respect proceed as before.

Consequently, if we make the present worth of £1 for the given time, at the given rate per cent., the ratio of a progression of which the number of terms is equal to the number of years the annuity has to continue; the sum of this series multiplied by the given annuity will be its present worth.

Thus to find the present worth of an annuity of £350 for 5 years, at 5 per cent., we shall have the series $\left(\frac{20}{21}\right)$: $\left(\frac{20}{21}\right)^2$: $\left(\frac{20}{21}\right)^3$: $\left(\frac{20}{21}\right)^4$: $\left(\frac{20}{21}\right)^5$; and the sum of this series multiplied by 350 will be the present worth of the given annuity, £350.

To find the sum of this series, we multiply the last term $\left(\frac{20}{21}\right)^5$, by the ratio $\frac{20}{21}$, and get the product $\left(\frac{20}{21}\right)^6$, from which subtracting the first term $\frac{20}{21}$, we have $\left(\frac{20}{21}\right)^6 - \frac{20}{21}$ $= \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} = \frac{64000000}{85766121} - \frac{81682020}{85766121} = \frac{17682020}{85766121}$.

Dividing this remainder by the ratio less 1, or $\frac{20}{21} - \frac{21}{21}$ $= -\frac{1}{21}$, we have $\frac{17682020}{85766121} \div -\frac{1}{21} = \frac{17682020}{85766121} \times -\frac{21}{1} = \frac{371322420}{85766121}$ the sum of the series; and multiplying this sum by 350, we have $\frac{371322420}{85766121} \times \frac{350}{1} = \frac{129962847000}{85766121}$ $=$ £1515 „ 6*s*. 4*d*., the present worth of the given annuity, £350; hence the

RULE.

1. Make the present worth of £1 for 1 year, at the given rate per cent., the ratio of a geometrical pro-

gression, of which the number of terms is equal to the number of years the annuity has to continue.

2. Find the sum of the series, and it will be the present worth of an annuity of £1 for the time; and multiply this sum by the given annuity, and the product will be the present worth required.

EXAMPLES FOR PRACTICE.

1. What is the present worth of an annuity of £100 to continue for seven years, discounting at 5 per cent., compound interest? *Ans.* £578 ,, 12*s.* 8¾*d.*

2. What is the present worth of an annuity of £215 for 10 years, at 4 per cent. compound interest? *Ans.* £1743 ,, 14*s.* 9¾*d.*

3. What ready money must be paid for the purchase of an annuity of £175 for 12 years, at 3 per cent., compound interest? *Ans.* £1741 ,, 19*s.*

4. What sum of money must be paid down as an equivalent for the discontinuance of an annuity of £30 ,, 15*s.* which had 17 years of its term unexpired, at 5 per cent.? *Ans.* £346 ,, 13*s.* 6½*d.*

5. What must be paid for the purchase of an annuity of £500 for 27 years, at 4 per cent., compound interest? *Ans.* £8164 ,, 15*s.* 9½*d.*

6. What is the present worth of an annuity of £700 for 50 years, discounting at 5 per cent., compound interest? *Ans.* £12779 ,, 2*s.* 10½*d.*

PROBLEM III.

To find the present worth of an annuity to continue for ever, or the price of a freehold estate, to commence immediately at compound interest. This problem is nothing more than to find a principal of which the annual interest at the given rate per cent. will be equal to the annuity itself.

And as in interest, the principals vary directly as the sums of interest, we have only to say, as the interest of £100 for 1 year at the given rate per cent. is to the annuity or yearly value of the estate, so is £100 to the present worth.

Thus, to find the present worth of a perpetual annuity or freehold estate of £150 per annum, at 5 per cent., we have as the interest 5 : 150 the annuity : : £100 : £3000, the present worth required.

EXAMPLES FOR PRACTICE.

1. What is the present worth of a perpetual annuity of £125 „ 10*s*. at 4 per cent.? *Ans.* £3137 „ 10*s*.

2. What ready money will purchase a freehold estate of £250 at 5 per cent.? *Ans.* £5000.

3. What is the present worth of a perpetual annuity of £75 „ 17*s*. 6*d*. at $4\frac{1}{2}$ per cent.? *Ans.* £1686 „ 2*s*. $2\frac{1}{4}$*d*.

4. What is the present worth of a perpetual annuity of £500 at 5 per cent.? *Ans.* £10000.

6. What ready money will purchase a freehold estate of £187 „ 10*s*., allowing the purchaser $3\frac{1}{2}$ per cent.? *Ans.* £5357 „ 2*s*. $10\frac{1}{4}$*d*.

6. What ready money must be paid for the purchase of a perpetual annuity of 150 guineas, at 4 per cent.? *Ans.* £918 „ 15*s*.

PROBLEM IV.

To find the present worth of a perpetual annuity, or of a freehold estate in reversion, at compound interest.

The present worth of an annuity in reversion will be less than its present worth, if to commence immediately, by the present worth of the same annuity for the time which is to elapse before it becomes payable.

Consequently, if we find the present worth of the annuity as to commence immediately, and subtract from it

the present worth of the same, for the time to elapse before it becomes payable, the remainder will be the present worth of the annuity in reversion.

Thus, to find the present worth of a perpetual annuity of £75, to commence at the end of 5 years at 5 per cent., compound interest, we have 1st, 5 : 75 :: 100 : £1500, the present worth, if it were to commence immediately.

To find the present worth of an annuity of £75, for 5 years, the time before it becomes payable, we have $\frac{20}{21} : \left(\frac{20}{21}\right)^2 : \left(\frac{20}{21}\right)^3 : \left(\frac{20}{21}\right)^4 : \left(\frac{20}{21}\right)^5$, and the sum of this series multiplied by 75 $= \frac{371322420 \times 75}{85766121}$ $= \frac{27849181500}{85766121} =$ £324 ,, 14*s*. 2¾*d*., the present worth for 5 years.

Lastly, subtracting this present worth from £1500, the present worth of the annuity as if commencing immediately, we have £1500 − £324 ,, 14*s*. 2¾*d*. = £1175 ,, 5*s*. 9¼*d*., the present worth of the same annuity to commence at the end of 5 years.

Or, if we find the present worth of £1500 for 5 years, at compound interest, we shall obtain the same result; for $\left(\frac{20}{21}\right)^5$ multiplied by 1500 $= \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} = \frac{3200000}{4084101}$ the last term of the series; and multiplying this by 1500, we have $\frac{3200000}{4084101} \times \frac{1500}{1} = \frac{4800000000}{4084101} =$ £1175 ,, 5*s*. 9¼*d*., the present worth of the annuity in reversion.

RULE.

Find the present worth of the annuity as if it were to commence immediately, and discounting this present worth at compound interest for the time to elapse before the annuity becomes payable, the present worth of this sum will be the present worth of the annuity in reversion.

EXAMPLES FOR PRACTICE.

1. What is the present worth of a perpetual annuity of £50, to commence at the end of 7 years, discounting at 4 per cent.? *Ans.* £950 „ 17*s.* 11½*d.*

2. What ready money must be paid for the purchase of a freehold estate of £75 in reversion after 9 years, allowing the purchaser 5 per cent. compound interest?

Ans. £966 „ 18*s.* 3¼*d.*

3. What is the present worth of an annuity of £150 „ 15*s.*, to commence after 15 years, discounting at 5 per cent., compound interest? *Ans.* £1450 „ 5*s.* 4½*d.*

4. What annual rent in reversion for 7 years may be purchased for £1500 at 6 per cent., compound interest?

Ans. £135 „ 6*s.* 6¼*d.*

MISCELLANEOUS EXAMPLES.

1. What sum must be paid down as an equivalent for adding 7 years to the term of a lease of a house rented at £50 per annum; reckoning compound interest at 5 per cent.? *Ans.* £289 „ 6*s.* 4¼*d.*

2. If an annuity of £50 for 25 years be increased to £75, in how many years should it be discontinued?

Ans. In 16$\frac{2}{3}$ years.

3. Which is most advantageous, for the tenant of a house on lease for 7 years at £70 rent, to pay £70 rent, with a premium of £200, or to pay £85 per annum without any premium?

Ans. The tenant will gain £77 „ 17*s.* 5*d.* by paying £85 per annum.

4. Suppose an estate to be leased for 21 years at £50 per annum, with a fine of £350 on entrance; what addition should be made to the rent as an equivalent for remitting the fine? *Ans.* £27 „ 5*s.* 11½*d.*

5. Which is most advantageous, a term of 15 years in an estate of £250 per annum, or the reversion of the estate

for ever after the expiration of that term, reckoning interest at 5 per cent.?

Ans. The term of 15 years is better than the reversion, by £189 „ 16*s.* 3¾*d.*

6. Which is the most advantageous, the lease of a house for 17 years at £30 per annum, with a premium of £500; at £45, with a premium of £300; or at £60, without any premium?

Ans. The lease at £60, without any premium, is better than at £45, with £300 premium, by £130 „ 17*s.* 9¼*d.*; and better than at £30, with £500 premium, by £161 „ 15*s.* 7½*d.*

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3

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51

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19

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72
~~20~~
4

~~101~~ 72 ~~36~~
4

$5\frac{2}{5}$

45

36 ~~107~~